Fluid Dynamics II PROBLEM SET 6 Due March 29, 2004

1. Derive the shock polar relation for oblique shocks,

$$v_2^2 = \frac{(u_1 u_2 - c_*^2)(u_1 - u_2)^2}{\frac{2}{\gamma+1}u_1^2 - u_1 u_2 + c_*^2}.$$

2. Suppose a steady 2D flow of a polytropic gas at Mach number 2, adjacent to a plane wall, encounters an abrupt bend of the wall by -30^o , producing an expansion fan. What is the Mach number downstream of this corner?

3. This is a problem for exploring the interesting phenomenon of sound generated by fluid flow (e.g. the bubbling of a brook or the sound of an air jet). (The pioneering work of Lighthill on this subject is surveyed by Ffowcs-Williams in Ann. Rev. Fluid Dyn. **1**, 197-222, which is on reserve.)

To study the sound generated aerodynamically in a fluid, assume that the density is $\rho = \rho_0 + \rho'$ and similarly for p, where ρ_0, p_0 are constants and $\rho' \approx p'/c_0^2$. For an inviscid gas, show that the equations of mass and momentum may then be combined to obtain a equation of the form

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = F.$$

Find the form of F in terms of ρ and $\mathbf{u}$ and their derivatives. What form is taken by F if we have $\rho \approx \rho_0$ and $\mathbf{u}$ is taken to have zero divergence? (See also L&L §75 for additional results.)