Applied Mathematics II **PROBLEM SET 3** **Due February 20, 2003**

1. Problem 1.4, page 34 of text.

2. Problem 2.7, page 41 of text. Gie the integral curves in parametric form. Show in part (b), for example, that the curves are given by

$$x = t + x_0, y = [y_0^3 + \frac{3}{4}(x_0^4 - (t + x_0)^4))]^{1/3}, z = z_0$$

where t is the parameter and the curves poass through the point x_0, y_0, z_0 when $t = 0$.

3.Solve the following problems for $u(x, y)$:

$$(a) \quad xu_x + y_y = 1, u(x, 0) = e^x.$$

$$(b) \quad xu_x + (y^2 + 1)u_y = u, u(x, 0) = e^x.$$

4. Solve the linear problem

$$u_t - tx^2 u_x = 0, \ u(x, 0) = x + 1,$$

and sketch the characteristic curves in the x, t-plane. Show that the solution becomes infinite on a certain curve in the x, t plane.

5. Solve the linear first-order equation

$$u_t + e^y u_x + u_y = 0, u(x, y, 0) = x + y.$$

First find the characteristic curves in the x, y-plane and give the functions $x(x_0, y_0, t$ and $y(x_0, y_0, t)$ which determine these curves. Then find $u(x, y, t)$.

6. Solve, using characteristics, the following problem for a quasilinear wave equation:

$$u_t + u^2 u_x = 0, \ u(x_0, 0) = \begin{cases} 0, & \text{if } x_0 < 0, \\ x_0, & \text{if } x_0 \geq 0. \end{cases}$$

Show that u is constant on a characteristic and that the characteristics are straight lines. Then find $u(x, t)$ as an explicit function of x, t for $t > 0$. Check your answer by differentiation.