**Evaluation of a a contour integral considered in lecture 6**

We saw that

$$\frac{\sqrt{\pi}(1+i)}{2\sqrt{2}} = \int_0^\infty e^{ix^2}dx = \int_0^\infty e^{iR^2 e^{i\pi/4}} e^{i\pi/8} dR = I$$

when we integrated from $(0,0)$ to $(0,R)$, then along $|z| = R$ to the ray $\theta = \pi/8$, then along that ray to (0,0), after letting the arc move off to infinity.

Now $e^{i\pi/8} = 2^{-1/4}\sqrt{1+i}$. Thus we have

$$I = \sqrt{1+i}\int_0^\infty e^{-t^2} eit^2 dt.$$

Thus

$$\int_0^\infty e^{-t^2} cos(t^2)dt = \sqrt{\pi/8}\Re\sqrt{1+i}.$$

But if $\sqrt{1+i} = a+ib$, the $1+i = a^2 - b^2 + 2iab$, $a^2 - b^2 = 1$, $2ab = 1$. Solving for $a > 0$ we get $a = \sqrt{\frac{1+\sqrt{2}}{\sqrt{2}}}$. Thus

$$\int_0^\infty e^{-t^2}\cos(t^2)dt = \frac{1}{4}\sqrt{\pi}\sqrt{1+\sqrt{2}}.$$