Lecture 1: Background and overview

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1 General remarks

This course will be devoted to primarily to natural locomotion in a dissipative fluid. Natural because we will be looking at the evolved mechanism of swimming and flying in the natural world, as opposed to solutions to transport devised by our technology. The role of dissipation is important and will be touched on below. Today I will not be getting into many technical details, but will try to give a sense of the scope of the course.

What do we mean by locomotion? Intuitively, it is what we do to get from point A to point $B \neq A$. *Forward flight* is certainly an example of locomotion in birds and insects. In flying we generally accept that the locomoter is not neutrally buoyant, so part of the locomotion is maintaining vertical position. We add that some fish use fins in a way similar to the wings of a bird, and so their motion might be termed flying as well. On the other hand passive motion, such as that obtained by varying the buoyancy, something marine animals can do, should not be considered locomotion proper, nor should diffusion of microscopic particles by Brownian motion.

Hovering of a heavy body will be considered as a kind of locomotion, even though the object does not to move at all! The reason for this is that a heavy hovering body delivers momentum downward to maintain position in a gravitational field. This downwash is a flow through which the hovering body can be said to "fly". Flying combines aspects of both hovering and swimming, in a sense. A kind of complementary problem is the pumping of fluid through a tube by the movement of the tube walls, so-called "valveless pumping". Many of the techniques models used in locomotion also apply to the study of pumping.

The models of locomotion we shall study will all involve time-dependent motions of the body. Thus the fluid dynamics is largely nonsteady, that is all variables depend on both space and time. While this is a source of some difficulty in the analysis, it gives rise to a rich set of interesting flow problems, only a few of which have been studied in detail.

2 Characteristic parameters

In our analysis of locomotion the most important physical parameters are: a length L typical of the locomoter; a speed U of locomotion, a frequency ω of the body movements causing the locomotion, the viscosity of the fluid, μ, and the fluid density, ρ. We shall be dealing with an incompressible Newtonian viscous fluid (to be defined mathematically next lecture), so sound speed will not be involved, and it turns out that only the ratio $\mu/\rho \equiv \nu$, called the *kinematic viscosity* arises in the dynamics. This parameter has dimensions $[\nu] = length^2/time$. From the parameters L, U, ω, ν there are then two distinct dimensionless parameters, where length and time dimensions cancel. The first and most important is the *Reynolds number* $Re \equiv UL/\nu$. This parameter measure the importance of inertial forces relative to viscous forces. When you abruptly move your hand through water, the force you feel is mainly inertial, associated with your accelerating the water. The Reynolds number of this movement is very large. (Estimate it yourself using $\nu \approx .01\ cm^2/sec$ for water.) But, if you move a spoon slowly through very thick honey, you are feeling mainly viscous forces and the value of Re is much less than 1.

The second dimensionless parameter is the *Strouhal number* $St \equiv \frac{\omega}{UL}$. This is a dimensionless frequency parameter whose importance can be appreciated only after we consider models of locomotion in more mathematical detail.

There is a third, derived parameter $Re_\omega \equiv \omega L^2/\nu = ReSt$, called the *frequency Reynolds number*. This parameter will also be of interest to us. The reason for considering Re_ω is that it is a Reynolds number based entirely upon intrinsic properties of the locomoter (L, ω) and fluid medium (ν). Re is dependent as well on the swimming speed U, which is not known *a priori*, and will generally depend on exactly how the locomoter changes its shape (by flapping wings or tail e.g.).

In the table which follows we estimate these parameters for various creatures:

Locomoter		*L(cm)*	*U (cm/sec)*	$\omega(sec^{-1})$	$UL/\nu = Re$	$\omega L/U = St$	$\omega L^2/\nu = Re_\omega$	*Remarks*
Stokesiam realm	*Bacterium*	10^{-5}	$10^{-2} - 10^{-3}$	10^4	10^{-5}	$10 - 10^2$	$10^{-3} - 10^{-4}$	*Limit of Navier-Stokes theory.*
	Spermatozoan	$10^{-2} - 10^{-3}$	10^{-2}	10^2	$10^{-2} - 10^{-3}$	$10 - 10^2$	10^{-1}	*Flag. diam.*$\approx 10^{-5} cm.$
	Ciliate	10^{-2}	10^{-1}	10	10^{-1}	1	10^{-1}	*cilium length*$\approx 10^{-3} cm.$
Intermediate realm	*Small wasp*	10^{-2}	10^{-1}	10	10^{-1}	1	10^{-1}	*U is wing speed hovering*
	Pteropod	.5	.5	1	25	1	10	*Flapping mode.*
Eulerian realm	*Locust*	4	400	20	10^4	.2	10^3	*Wing Re* $approx 2000$
	Pigeon	25	$10^2 - 10^3$	5	10^5	.25	10^4	*Wing Re* $\approx 10^4$
	Fish	50	100	2	5×10^4	1	10^4	

If gravity is important, as in the wave riding of dolphins, another dimensionless parameter appears because of the introduction of the acceleration of gravity, g, with dimensions $[g] = length/time^2$. This is the *Froude number* $F \equiv \frac{U^2}{Lg}$. Gravity plays no role if the locomoter is neutrally buoyant, and in problems of flight determines the relative weight of the body.

Finally, other parameters arise if there are interactions between locomoters, as in the schooling of fish, formation flight of birds, swarming of insects, and the interesting phenomenon of bioconvection of microorganisms.

Observe in the above table that the range of Reynolds numbers is enormous, much greater than that of the Strouhal number. That is something one would like to understand from the mechanics of swimming and flying. In fact, in the forward flight of birds and insects one sees almost universally that St is in the range .2-.4. This strongly suggests that these values are in some way associated with efficient flight, given the high energetic cost of flapping flight. This is the kind of thing we can hope to understand through analysis of locomotion from first principles.

I have emphasized the dissipation of the fluid and have divided the table into a *Stokesian realm* where Re is small and viscous forces large, an *Eulerian realm* where Re is large and the viscous forces *nominally* small, and an *Intermediate realm* in between. In the Stokesian realm, the viscous dissipation is large, and momentum is diffused almost instantaneously. In the mathematical limit of zero Re, this has a remarkable consequence: *At each instant of time, the position of fluid particles is a unique function of the instantaneous body position.* Thus as the body moves though a path of shapes, all points of the fluid external to the body move simultaneously along paths, and if the path through shapes is reversed and the body returns to its starting position, so do all fluid particles.

Consider now a *cycle* of shape changes, with starting and ending shapes the same, so that we have a closed loop in "shape space". Suppose that the cycle is *reciprocal* in the sense that the sequence of shapes is invariant under time reversal. It then follows that the body cannot locomote.

The “proof” is as follows: As shapes change through a forward cycle, let the body move from point A to point $B \neq A$. Now reverse time and go pack through the cycle. The body must then move from B to A. But this is impossible since the motion is assume to be reciprocal and the sequence is the same under time reversal, i.e. it actually must move to $B + (B - A) = 2B - A$ thinking of A, B as position vectors. If $2B - A = A$ we have $A = B$, so in fact there is no locomotion at all. This result is often called the *Scallop Theorem* since the opening and closing of a scallop as it propels itself is a reciprocal motion. (It follows that scallops do not locomote in the Stokesian realm!)

You might have noticed that I am here using time as a parameter, not as a dynamic variable. It is unimportant (within the limits of low Re theory) how fast the scallop may open of close. If it opens fast and closes slowly, or the opposite, the result is the same- no locomotion at $Re = 0$. All that matters is the sequence of shapes, not the timing of the changes.

Nature has devised a rich array of nonreciprocal shape changes to propel organisms in the Stokesian realm. Most depend upon slender organelles, flagella or cilia, moving in a nonreciprocal manner. We will study how locomotion is accomplished in these cases, and see that it depends upon the fact that a slender body has, in a rough sense, a local resistance to transverse motion (motion perpendicular to the line of the body) which is about twice that of parallel motion (along the line of the body).

In the Eulerian realm, I have noted that viscosity is “nominally” not important. This is not to say that the effects of viscosity are negligible, however, since in localized regions such as *boundary layers* viscous stresses cannot be neglected, even to first approximation, in the fluid dynamics. We deal with a continuous fluid, and as long as there is some viscosity, the fluid will adhere to a solid surface. The fluid at the surface must move with the surface. If one takes the mathematically drastic step of setting $\nu = 0$, it become possible physically for the fluid to simply slip over a surface, so that the only condition at a solid wall must be that fluid cannot penetrate into the wall, with no constraint on tangential components.

If $\nu = 0$ identically we say that the fluid is *ideal* or *inviscid*. The simplest inviscid motions have the property that they are *irrotational*. To define this term, let $\mathbf{u}(\mathbf{x}, t)$ be the velocity field of the fluid. (This is an *Eulerian* field, a distinction I shall try to clarify next week.) The *vorticity field* is then $\omega = \nabla \times \mathbf{u}(\mathbf{x}, t)$. An irrotational flow has $\omega = 0$, and so $\mathbf{u}(\mathbf{x}, t) = \nabla\phi$ with ϕ the potential of the flow.

Now the fluid dynamics of inviscid flow establishes the following:

• If a body moves steadily through an inviscid fluid which is at rest at ∞, then the vorticity must vanish everywhere, and the flow is a potential flow.

• If a regular potential flow exists in the exterior of a body moving with speed U, then the force on the body is zero, and no work need be done to keep the body in motion.

Thus, in this inviscid setting, the locomotion problem becomes trivial. A body once in motion will just keep moving. (I should mention that the *moment* acting on the body need not vanish, so a free object will tumble in general.

In fact of course, this effortless motion of a body is not what we observe. Due to the small dissipation present even at when $Re \gg 1$ work must be done to keep a body in motion. And inevitably, vorticity is injected into the fluid, by the process of *vortex shedding* The shedding of vorticity is enhanced at sharp edges, hence the organelles we see in Nature such as wings and fins.

The Intermediate realm is of interest as a zone of transition between the Stokesian and Eulerian locomoters. It is a regime where one can attempt to understand how Eulerian principles might have developed out of the Stokesian realm as size increased (given that life starts small, hence at low Re). In the table above I listed a *pteropod*, which is a small marine mollusc. The pteropod *Clione antarctica*, in its juvenile stage, is about 4 mm long and has three bands of cilia encircling a tube-like body. It is thus able to swim as a ciliate in the Stokesian realm. But it also possesses two small wings which it can extend and flap like a bird. These organelles are thus suitable for Eulerian swimming. The transition from one to the other has been studied as the size of the pteropods increased, and the increase of utility of wings relative to cilia observed. One of the problems we shall look at is how Eulerian locomotion can be viewed as arising from a mathematical bifurcation with respect to frequency Reynolds number, a bifurcation which occurs in the Intermediate realm.

Note: Some films illustrating this bifurcation in a numerical experiment were shown in the class. Anyone not present then who would like to see them should contact me.