Complex Variables I

Some problems for Lecture 1

September 5, 2006

Not to be handed in

- 1. Exercises pp. 4-5: 2,3,8,9.
- 2. Exercises pp. 13-14: 4,7,8,16.
- 3. Show that

$$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z_2}).$$

4. Show that

$$Re\left(\frac{z-1}{z+1}\right) = \frac{|z|^2 - 1}{1 + 2Re(z) + |z|^2}.$$

5. Verify by direct calculation that

$$\overline{\left(\frac{z}{z^2+1}\right)} = \frac{\overline{z}}{\overline{z}^2+1}.$$

- 6. Exercises pp. 21-22: 1,5,7
- 7. Given that $z = re^{i\theta}$, prove by induction on n that $z^n = r^n e^{in\theta}$. (See page 20)