An example of a non-smooth contour

The condition that $|z'(t)| \neq 0$ can be thought of as saying the "speed" of motion of a point along the contour (t then being time) does not vanish. When the speed vanishes we can get sharp corners. The simplest example I can think of is $z(t) = t^3 + it^2$ at $t = 0$. Then $z' = 3t^2 + i2t$ and so $|z'(t)|$ vanishes there. For $t > 0$ we have $y = x^{2/3}$ and for $t < 0$ we have $y = (-x)^{2/3}$, so that $y = |x|^{2/3}$ in the vicinity of $z(0) = 0$. The contour thus has a cusp there, and is not "smooth". So long as $z' \neq 0$ the tangent to the curve changes continuously, which can be thought of as the definition of "smooth".