Derivation of scaling of the lower deck

$u_x + v_y = 0$ Gives

$$U/X = V/Y, \quad (1)$$

uu_x balancing $\frac{1}{R}u_{yy}$ gives

$$RU = X/Y^2, \quad (2)$$

.

In these the capital letters (except for R) represent scaling factor order relative to R.

The main deck supplies the pressure of order V/U, so we have from balancing p_x and uu_x

$$V = U^3, \quad (3)$$

.

Finally, the upstream boundary condition is that u match with a boundary layer, for which $\partial u/\partial y \sim R^{1/2}$. Thus

$$U^2 = Y^2 R, \quad (4)$$

Now from (1) and (3),

$$U^2 = Y/X, \quad (5)$$

From (4) and (5)

$$XYR = 1, \quad (6)$$

From (5) and (2),

$$R^2 Y^5 = X^3 \quad (7)$$

Solving (6) and (7) for X, Y as function of R, we obtain

$$X = R^{-3/8}, y = R^{-5/8}$$

Wev then see that

$$U = R^{-1/8}, V = R^{-3/8}, P = R^{-1/4}$$

in the lower deck.