Intro. to Math. Modeling   PROBLEM SET 3   Due February 7, 2005

1. Problem2 37.2 of text.

2. Problem 37.5 of text.

3. Prove that the *rate* of growth or decay of the population in the continuous logistic model is *monotonic*, that is $d^2N/dt^2 > 0$ or < 0 for all $t > 0$, provided that $N(0) > K/2$. (Hint: differentiate both sides of the equation $dN/dt = rN(1 - N/K)$ with respect to t.)

4. For some organisms the effective growth ($N^{-1}dN/dt$ is highest at some finite value b of the population. For example, mates may be hard to find at a low population, but there is too much competition for food at higher populations. A model of this situation is

$$\frac{dN}{dt} = N(r - (N - b)^2),$$

where r, b are positive constants.

(a) What are the feasible (i.e. positive) equilibrium populations for this model when $b^2 > r$ (case 1)? When $b^2 < r$ (case 2)?

(b) Classify as stable or unstable the feasible equilibria for the two cases in (a).

(c) For each case, indicate the equilibria on the N-axis of the phase plane and indicate the movement with time along the axis, as we did for the logistic equation. Verify that this movement is in accord with the classification of (b).

5.

(a) Show that the equilibrium $x_e = 1 - 1/r$ for the discrete logistic equation $x_{m+1} = rx_m(1 - x_m) = F(x_m)$ is stable when $1 < r < 3$.

(b) The expression F(F(x))=x, whose roots are the equilibrium solutions, may be written

$$rx(x - 1 + 1/r)(r^2x^2 - (r^2 + r)x + 1 + r) = 0.$$

Show that two new feasible roots (between 0 and 1) occur for $r > 3$. Verify that for r=3.1 these give the computed period-2 orbit .7645665, .5580141.