KNOWN MISPRINTS
"POSITIVE SCALAR CURVATURE WITH SKELETON SINGULARITIES"

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(1) **Lemma 6.1**: The bound for $h(t)^{\frac{n-2}{n-1}}$ in [LM19, (6.6)] and the replacement argument in [LM19, (6.7)] require that one ensure that the area-minimizing current one gets from the Euclidean isoperimetric inequality is homologous to $\Omega \cap \partial B_t(\mathbf{0})$ in $\mathbf{R}^n \setminus B_1(\mathbf{0})$, in order to offer a competitor in $\mathscr{C}$. This was not justified in the published paper but is, indeed, true for a.e. $t \geq t_0(n, \Lambda)$. All subsequent inequalities in [LM19, Lemma 6.1] are then to be taken for $t \geq t_0(n, \Lambda)$. The rest of the argument is unchanged. We are grateful to Prof. L.-F. Tam for pointing out the gap in the original argument and for insightful discussions.

The argument goes as follows. Denote by N_t the Euclidean area-minimizing current with boundary Σ_t, assuming t is among the full-measure set of t where Σ_t is a finite-mass integral current.

Claim 1: Every point $\mathbf{x} \in N_t$ satisfies $\operatorname{dist}_\delta(\mathbf{x}, \partial N_t) \leq d_0(n, \Lambda)$.

Set $d := \operatorname{dist}_\delta(\mathbf{x}, \partial N_t)$. By the Euclidean monotonicity formula for N_t, centered at $\mathbf{x} \in N_t$, we get $\mathcal{H}^{n-1}_\delta(N_t) \geq \omega_{n-1} d^{n-1}$. Also, $\mathcal{H}^{n-1}_\delta(N_t) \leq \mathcal{H}^{n-1}_\delta(\partial\Omega \cap B_t(\mathbf{0}))$ since N_t has optimal Euclidean area and the same boundary as $\partial\Omega \cap B_t(\mathbf{0})$. So,

$$\omega_{n-1} d^{n-1} \leq \mathcal{H}^{n-1}_\delta(\partial\Omega \cap B_t(\mathbf{0})) \leq \mathcal{H}^{n-1}_\delta(\partial\Omega) \leq c_1'(n, \Lambda)$$

by [LM19, (6.5)], $\Lambda^{-1}\delta \leq g$.

Claim 2: For $t \geq t_0(n, \Lambda)$, N_t is homologous to $\Omega \cap \partial B_t(\mathbf{0})$ in $\mathbf{R}^n \setminus B_1(\mathbf{0})$.

First, $\mathcal{H}^{n-1}_\delta(N_t) \leq \mathcal{H}^{n-1}_\delta(\partial\Omega \cap B_t(\mathbf{0}))$ as in *Claim 1*. Second, $\partial B_t(\mathbf{0})$ is Euclidean outer-minimizing, so $\mathcal{H}^{n-1}_\delta(\Omega \cap \partial B_t(\mathbf{0})) \leq \mathcal{H}^{n-1}_\delta(\partial\Omega \setminus B_t(\mathbf{0}))$. Thus:

$$\mathcal{H}^{n-1}_\delta(N_t) + \mathcal{H}^{n-1}_\delta(\Omega \cap \partial B_t(\mathbf{0})) \leq \mathcal{H}^{n-1}_\delta(\partial\Omega) \leq c_1'(n, \Lambda).$$

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Next, note that $N_t - (\Omega \cap \partial B_t(\mathbf{0})) = \partial\mathcal{U}$ for a domain $\mathcal{U} \subset \mathbf{R}^n$, since N_t is homologous to $\Omega \cap \partial B_t(\mathbf{0})$ in $\mathbf{R}^n$. (We want $\mathcal{U} \cap B_1(\mathbf{0}) = \emptyset$.) If $t \geq 2d_0(n, \Lambda) + 1$, then $N_t \cap B_{t/2}(\mathbf{0}) = \emptyset$ by *Claim 1*, so by the constancy theorem either $\mathcal{U} \cap B_{t/2}(\mathbf{0}) = \emptyset$ or $B_{t/2}(\mathbf{0}) \subset \mathcal{U}$. We are done if $\mathcal{U} \cap B_{t/2}(\mathbf{0}) = \emptyset$. Assume not. Since $\partial B_{t/2}(\mathbf{0})$ is Euclidean outer-minimizing and $B_{t/2}(\mathbf{0}) \subset \mathcal{U}$,

$$\begin{aligned} \mathcal{H}^{n-1}_\delta(\partial B_{t/2}(\mathbf{0})) &\leq \mathcal{H}^{n-1}_\delta(\partial\mathcal{U}) \\ &\leq \mathcal{H}^{n-1}_\delta(N_t) + \mathcal{H}^{n-1}_\delta(\Omega \cap \partial B_t(\mathbf{0})) \leq c_1'(n, \Lambda), \end{aligned}$$

a contradiction for all $t \geq t_0(n, \Lambda)$ sufficiently large.

(2) **Lemma 6.1**: In the chain of inequalities following [LM19, (6.7)], the coarea formula should be invoked with respect to the Euclidean metric and the Euclidean distance function $\operatorname{dist}_\delta(\mathbf{0}; \cdot)$. By $\Lambda^{-1}\delta \leq g \leq \Lambda\delta$, this changes c_3^{-1} to $(c_3')^{-1}$, for $c_3' = c_3(n, \Lambda)$, in the final inequality of the chain.

References

[LM19] Chao Li and Christos Mantoulidis. Positive scalar curvature with skeleton singularities. *Math. Ann.*, 374(1-2):99–131, 2019.