Linear Algebra

Problem set # 2

Sep 28, 2018

Due: Oct 4, 2018

1. Let $\sum_{j=1}^{n} t_{ij} x_j = u_i$, $i=1,\dots,m$, be an overdetermined set of linear equations — i.e. $m > n$. Take the case that in spite of this overdeterminacy, this system of equations has a solution, for some given and fixed $u_1,\dots,u_m$, and that this solution is unique. Show that it is possible to select a subset of n of these equations which uniquely determine the solution.

2. Let S be rotation around the x_1 axis by $90°$ in $\mathbb{R}^3$ and let T be rotation around the x_2 axis by $90°$ in $\mathbb{R}^3$. Show that $ST \neq TS$.

3. Show that $(ST)' = T'S'$, $(T+R)' = T'+R'$ and $(T^{-1})' = (T')^{-1}$, wherever meaningful. Note: for the 3rd equality you must show that T is invertible iff T' is invertible.

4. Describe explicitly a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ which has as its range the subspace spanned by $(1,0,-1)$ and $(1,2,2)$.

5. Let V be an n-dimensional vector space over a field K and let T be a linear transformation from V into V such that the range and null space of T are identical. Prove that n is even and give an example of such a linear tranformation T.

6. Let V be a vector space and $T \in \mathcal{L}(V, V)$. Prove that the following statements are equivalent:

(a) $R_T \cap N_T = \{0\}$

(b) If $T(Tx) = 0$, then $Tx = 0$.

7. Let $T \in \mathcal{L}(X, X)$ for some vector space X, $\dim X < \infty$. Suppose $\dim R_T = \dim R_{T^2}$. Show that $R_T \cap N_T = \{0\}$.