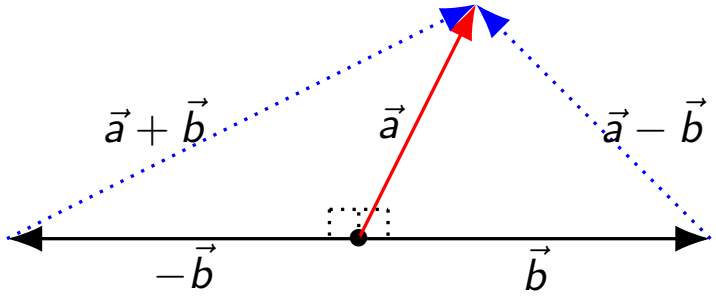
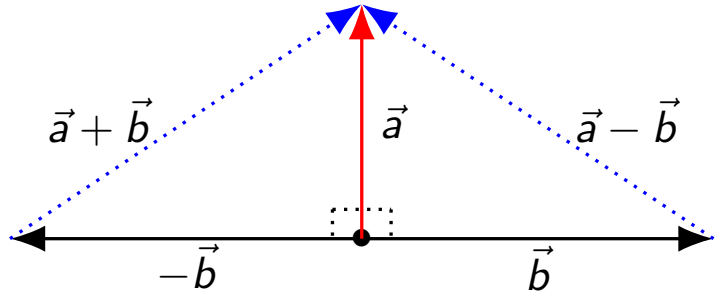
Dot Product Via Length and Orthogonality





- Two nonzero vectors $\vec{b} = \langle b_1, b_2, b_3 \rangle$ and $\vec{a} = \langle a_1, a_2, a_3 \rangle$ are **orthogonal**, if the angle from $\vec{a}$ to $-\vec{b}$ is equal to the angle from $\vec{b}$ to $\vec{a}$.
- By SAS, this holds if and only if the triangle with sides $-\vec{b}$ and $\vec{a}$ (reflected about $\vec{a}$) is congruent to the triangle with sides $\vec{b}$ and $\vec{a}$
- By SSS, this holds if and only if

$$\begin{aligned} 0 &= |\vec{a}+\vec{b}|^2 - |\vec{a}-\vec{b}|^2 \\ &= |\langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle|^2 - |\langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle|^2 \\ &= 4(a_1 b_1 + a_2 b_2 + a_3 b_3). \end{aligned}$$