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The problem of weather prediction, considered from the viewpoints of mechanics and physics

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Superscript numbers indicate original footnotes (translated at the bottom of the page), E... numbers indicate editorial endnotes (at the end of the article), square brackets[] indicate editorial comments in the text.

If it is true, as any scientist believes[E1], that subsequent states of the atmosphere develop from preceding ones according to physical laws, one will agree that the necessary and suff cient conditions for a rational solution of the problem of meteorological prediction are the following:

1. *One has to know with suff cient accuracy the state of the atmosphere at a given time.*
2. *One has to know with suff cient accuracy the laws according to which one state of the atmosphere develops from another.*

I.

It is the task of observational meteorology to determine the state of the atmosphere at agreed-upon, suitable times. This task has not been solved to the extent necessary for rational weather prediction. Two gaps are particularly critical. Firstly, only land-based stations participate in the daily weather service. There are still no observations made at sea for the purpose of daily weather analysis, although the sea accounts for four f fths of the Earth's surface and therefore must exert a dominant inf uence. Secondly, the observations of the regular weather service are made at ground level only, and all data about the state of higher layers of the atmosphere are missing.

However, we have the technical means that will enable us to f ll these two gaps. By means of radiotelegraphy, it will be possible to include among the reporting stations steamships with f xed routes. And due to the great strides that aeronautic meteorology[E2] has made in the past years, it will no longer be impossible to get daily observations from higher atmospheric layers, both from f xed land measurement stations as well as from stations at sea.

We can hope, therefore, that a time will soon come when a complete diagnosis of the state of the atmosphere will be available, either daily or for specif ed days. The f rst condition for weather predictions according to rational principles will then be satisf ed.

II.

The second question then arises as to what extent we know, with suff cient accuracy, the laws according to which one state of the atmosphere develops from another.

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Atmospheric processes are of a mixed mechanical and physical nature. For every single process, we can propose one or several mathematical equations derived from mechanical or physical principles. We will possess suff cient knowledge of the laws according to which atmospheric processes develop if we can write down as many equations independent from one another as there are unknown quantities. The state of the atmosphere at any point in time will be determined meteorologically when we can calculate velocity, density, pressure, temperature and humidity of the air for any point at that particular time. Velocity is a vector and therefore represented by three scalar variables, the three velocity components, which means that altogether, there are seven unknown parameters to be calculated.

For calculating these parameters, we can propose the following equations:

1. The three hydrodynamic equations of motion. These are differential relations among the three velocity components, density and air pressure.

2. The continuity equation, which expresses the principle of the conservation of mass during motion. This equation is also a differential relation, namely between the velocity components and the density.

3. The equation of state for the atmosphere, which is a f nite relation among density, air pressure, temperature, and humidity of a given air mass.

4. The two fundamental laws of thermodynamics, which allow us to write two differential relations that specify how energy and entropy of any air mass change in a change of state. These equations do not introduce any new unknowns into the original problem because energy and entropy are expressed by the same variables that appear in the equation of state and relate the changes of these quantities to changes of other known parameters. These other quantities are, f rstly, the work done by an air mass, which is determined by the same variables that appear in the dynamical equations; secondly, the heat quantities received from or given off to the outside. These heat quantities will be constrained by physical data on incoming and outgoing radiation, and on the warming of the air in contact with the Earth's surface [conduction].

It should be emphasized that the problem is considerably simplif ed if there is no condensation or evaporation of water and thus water vapour contained in the air can be considered a constant constituent. The problem will then have one variable less and one equation can be eliminated, namely the one that bases on the second law of thermodynamics. On the other hand, if we had to include several variable components of the atmosphere, then the second law of thermodynamics would result in a new equation for each new constituent.

We can therefore set up seven equations independent from each other with the seven normally occurring variables. As far as it is possible to have an overview of the problem now, we must conclude that our knowledge of the laws of atmospheric processes is suff cient to serve as a basis for a rational weather prediction. However, it must be admitted that we may have overlooked important factors due to our incomplete knowledge. The interference of unknown cosmic effects is possible. Furthermore, the major atmospheric phenomena are accompanied by a long list of side effects, such as those of an electrical and optical nature. The question is to what extent such side effects could have considerable effects on the development of atmospheric processes. Such effects evidently do exist. The rainbow, for instance, will result in a modif ed distribution of incoming radiation and it is well known that electrical charges inf uence condensation processes. However, evidence is still lacking on whether processes of this kind have an impact on major atmospheric processes. At any rate, the scientif c method is to begin with the simplest formulation of the problem, which is the problem posed above with seven variables and seven equations.

III.

Only one of the seven equations has a f nite form, namely the equation of state. The other six are partial differential equations. By means of solving the equation of state, one of the seven unknowns can be eliminated. The task then consists of integrating a system with six partial differential equations with six unknowns by using the initial conditions that are given by the observations of the initial state of the atmosphere.

There can be no question of a strictly analytical integration of the system of equations. As is well known, calculating the motion of three points that inf uence each other according to a law as simple as Newton's already far exceeds the means of today's mathematical analysis. There is evidently no hope of knowing the movements of all points of the atmosphere which are inf uenced by much more complicated interactions. However, the exact analytical solution would still not result in what we need, even if we could write it down. In order to be of practical use, the solution must primarily have a clear form and therefore, numerous details have to be neglected which would have had to be contained in an exact solution. The prediction may thus ref ect only mean conditions over long distances and for extended time intervals. This can be, for instance, between two meridians and for hourly steps, but not from millimetre to millimetre or from second to second.

Therefore, we abandon any thought of analytical integration methods and think of the problem of weather prediction in the following practical form:

Based upon the observations that have been made, the initial state of the atmosphere is represented by a number of maps that show the distribution of the seven variables from layer to layer in the atmosphere. With these maps as a starting point, new maps of a similar kind should be drawn that represent the new state of the atmosphere from hour to hour.

Graphical or mixed graphical and numerical methods are required to solve the task. These methods must be derived either from the partial differential equations or from the dynamical-physical principles that form the basis of the equations. There is no reason to doubt that elaborating such methods is possible. Everything will depend on succeeding in breaking down the problem, which is overwhelmingly difficult as a whole, into partial problems of which none represents insurmountable difficulties.

IV.

In order to achieve this partitioning into partial problems, we have to apply the general principle that forms the basis of infinitesimal calculus of several variables. For computational purposes, the simultaneous variation of several variables can be replaced by sequential variations of individual variables or of individual groups of variables. If one goes to the limit of infinitesimal intervals, the approach corresponds to the exact methods of infinitesimal calculus[E3]. If finite intervals are used, the method is close to that of the finite difference and of the mechanical quadrature[E4], which we will have to use here.

However, this principle must not be used blindly, because the practicality of the method will mainly depend on the natural grouping of the variables, so that both mathematically and physically well-defined and clear partial problems will result. Above all, the first decomposition will be fundamental. It must follow a natural dividing line in the main problem.

Such a natural dividing line can be specified. It follows the boundary line between the specifically dynamic and the specifically physical processes, of which atmospheric processes are composed. The decomposition along this boundary line results in a partitioning of the main problem into purely hydrodynamic and purely thermodynamic partial problems.

The link that connects the hydrodynamic and the thermodynamic problems can very easily be cut, indeed, it can be cut so easily that theoretical hydrodynamicists have always done so in order to avoid any serious contact with meteorology; because the link is given by the equation of state. If one assumes that this equation does not contain temperature and humidity, the equation corresponds to the "supplementary equation" normally used by hydrodynamicists, which is a relation only between density and pressure. Thereby, fluid motions are studied under circumstances where any explicit consideration of the thermodynamic processes drops out.

Instead of completely omitting temperature and humidity from the equation of state, we can regard them as given quantities for shorter time intervals, using values that are either given by observations or preceding calculations. After the dynamical problem has been solved for this time interval, new values of temperature and humidity can be calculated according to purely thermodynamic methods. These again are regarded as given quantities when the hydrodynamic problem for the next time interval is solved and so on.

V.

The general principle for the first decomposition of the main problem is thus given. The practical procedure offers the choice between several different approaches, depending on the method by which the hypotheses about temperature and humidity are introduced. However, it does not make sense to look closer into this in a general discussion such as this one.

The next major question will be, to what extent the hydrodynamic and the thermodynamic partial problems can be individually solved in a sufficiently simple way.

We will first consider the hydrodynamic problem, which is the principal one, since the dynamic equations are the true prognostic equations. It is due only to them that time is introduced as an independent variable into the problem; the thermodynamic equations do not contain time.

The hydrodynamic problem will suit perfectly for graphical solutions. Instead of calculating with three dynamic equations, one can execute simple parallelogram constructions for an adequate number of selected points. The regions in between are complemented by graphic interpolation or visual judgement. The main difficulty will lie in the constraints to motion that follow from the continuity equation and the boundary conditions. However, the test of whether or not the continuity equation is satisfied can also be made with graphical methods. In so doing, the topography can be taken fully into consideration by carrying out the construction on maps which represent the topography in a usual way.

The solution of the hydrodynamic partial problem will therefore not pose any major mathematical difficulties. However, there is a considerable gap in knowledge of the factors that we must take into account, namely in our very incomplete knowledge of frictional stress in the atmosphere. While meteorologists are forced to deal with the mean movement of large air masses, friction depends on very small-scale velocity differences. Thus, it is not possible to apply the frictional terms of the hydrodynamic equations by using the coefficient of friction established in laboratories. Rather, empirical results on the effective resistance to the movement of large air masses have to be taken into consideration. Sufficient data of this kind is already available for undertaking first experiments in forecasting the airflow, and in time, these experiments will provide the necessary corrections and completions.

From a mathematical point of view, the thermodynamic partial problem is significantly simpler than the hydrodynamic. From the solution to the hydrodynamic problem, we obtain the work performed by the air masses during their displacements. With the knowledge of this work and the knowledge of the heat supplied by incoming radiation and emitted by outgoing radiation, we can calculate the new temperature and humidity distribution according to thermodynamic principles. In mathematical terms, the calculations will not be more difficult than similar calculations in laboratory experiments, where air masses are at rest in a closed space. Extensive preliminary work is also provided by the studies of HERTZ, VON BEZOLD and others.[E5]

As in the hydrodynamic problem, the main difficulty will be our fragmentary knowledge of various factors, with which this calculation must be carried out. In the beginning, the estimates of the heat quantities that the air masses contain due to the difference between incoming and outgoing radiation, and of the water quantities that evaporate on the sea surface or fall in the form of rain from the clouds, will be very uncertain. However, we have sufficient knowledge for attempting first calculations, and more exact values of the constants will result from continued work. They will refer to the different countries and oceans, different heights in the atmosphere, different weather conditions, different degrees of cloud density and so on.

VI.

It is certain that there will be no insurmountable mathematical difficulties in the approach described.

After the graphical methods are elaborated on and at hand and after the necessary tabular aids have been assembled, the individual operations will probably also turn out to be easily implementable. Furthermore, the number of single operations need not be excessively large. The number will depend on the length of the time intervals for which the dynamical partial problem is solved. The shorter the fixed time intervals are chosen, the more complicated the work will become, but also the more accurate the result will be. The longer the fixed time intervals are chosen, the faster the target will be achieved, but at the cost of accuracy. Only by experience, final results as to the adequate choice can be given. Intervals of one hour should usually be adequate even if high accuracy is aimed at, because only in exceptional circumstances will air masses travel further than one degree of longitude within one hour, and only in exceptional circumstances will their tracks curve more strongly within this time. Therefore, the conditions for using the parallelogram construction with straight lines are fulfilled. When one has gained enough experience and has thereby learned to use instinct and visual judgement, it would probably be possible to work with much longer time intervals such as six hours. A 24-hour weather prediction would then require doing the hydrodynamic construction four times and calculating the thermodynamic correction of temperature and humidity four times.

It might therefore be possible that at some time in the future, a method of this kind will form the basis of a practical, daily weather prediction. However this will evolve, sooner or later the fundamental scientific study of atmospheric processes according to methods based on mechanical and physical laws will have to be started. And thereby, one will necessarily come across a method similar to that just outlined.

Having acknowledged this, a general plan for the dynamical-meteorological research is given.

The main task of observational meteorology will be to provide simultaneous observations of all parts of the atmosphere, at the Earth's surface and aloft, over land and over sea.

Based on the observations made, the first task of theoretical meteorology will then be to derive the clearest possible picture of the physical and dynamical state of the atmosphere at the time of the observations. This picture must be in a form that is appropriate to serve as a starting point for a weather prediction according to rational dynamical-physical methods.

This first introductory task will not be a minor one, since it is evidently much more complicated to represent the state of the atmosphere not just at sea level, as it is done now, but at all heights. Additionally, direct observation of the higher layers of the atmosphere will always remain limited. Therefore, observations of the higher layers of the atmosphere must be used to the utmost. From the directly observable quantities we must calculate as comprehensively as possible all accessible data on the non-observable ones. For that purpose one has to utilize the relationships between the different quantities. Thus, even for constructing a coherent picture of the state of the atmosphere from scattered observations, one must extensively use dynamical-physical methods.

Eventually, the second and most challenging task of theoretical meteorology will be to construct the pictures of the future states of the atmosphere from the picture of the current state of the atmosphere as a starting point, either according to the method outlined here, or according to a method of a similar kind. The comparison of the predicted fields with those constructed later from observations will, on the one hand, enable a general validation of the correctness of the method and, on the other hand, provide conclusions leading to better values of the constants and indications regarding improvements of the method.

I will later on return to the various main points of the procedure described.

Editorial endnotes

E1 This is not the first translation of BJERKNES' seminal paper. Another translation can be found at

www.history.noaa.gov/stories_tales/bjerknes.html. A very nice and highly recommended translation is that by Y. MINTZ, Los Angeles, 1954, published in:
SHAPIRO, M.A., S. GRØNAS, 1999: The Life Cycle of Extratropical Cyclones, American Meteorological Society, Boston, 1–4.
Our translation is similar to the latter (on some occasions we follow MINTZ's translation) but is generally somewhat closer to the German original and is supplemented with editorial endnotes according to our "Classic paper" style.
The above mentioned volume also contains a number of other interesting papers relating to Bjerknes and his workh:
ELIASSEN, A., 1999: Vilhelm Bjerknes' early studies of atmospheric motions and their connection with the cyclone model of the Bergen School. – In: SHAPIRO, M.A., S. GRØNAS (Eds): The Life Cycle of Extratropical Cyclones, American Meteorological Society, Boston, 5–13.
VOLKERT, H., 1999: Components of the Norwegian cyclone model: Observations and theoretical ideas in Europe prior to 1920. – In: SHAPIRO, M.A., S. GRØNAS (Eds): The Life Cycle of Extratropical Cyclones. American Meteorological Society, Boston, 15–28.

E2 Aerology experienced an impressing development in the years around 1900. Aerological observatories were founded for instance in the United States, France, Russia, and Germany using kites, tethered balloons, and registering balloons. Kite ascents were also performed from ships. The International Aeronautic Commission (chaired by Hugo HERGESELL) that was formed within the International Meteorological Committee coordinated the soundings. Simultaneous ascents ("International Aerological Days") were performed and were considered by some as the start of a new era of synoptic aerology (see HOINKA, 1997).
HOINKA, K.P., 1997: The tropopause: Discovery, def nition and demarcation. – Meteorol. Z. **6**, 281–303.

E3 "Inf nitesimal calculus" in this context refers to differential calculus.

E4 "Mechanical quadrature" is a numerical integration method.

E5 Heinrich Rudolf HERTZ (1857–1894), German physicist who is well known for his electromagnetic research (SI unit Hertz). His interest in meteorology is related to his contacts with VON BEZOLD (see below). HERTZ (who was a teacher of Vilhelm BJERKNES) developed the emagram, the f rst thermodynamic diagram.
Johann Friedrich Wilhelm VON BEZOLD (1837–1907), German meteorologist and atmospheric thermodynamicist, 1885–1907 director of the Prussian Institute of Meteorology at the University of Berlin. He worked on (and indeed introduced the concepts of) potential temperature, equivalent temperature and pseudo-adiabatic processes.
Two often cited publications of these two scientists are:
VON BEZOLD, W. 1888: Zur Thermodynamik der Atmosphäre. Zweite Mittheilung. Potentielle Temperatur. Verticaler Temperaturgradient. Zusammengesetze Convection. – Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin. Jahrgang 1888, 1189–1206.
HERTZ, H., 1884: Graphische Methode zur Bestimmung der adiabatischen Zustandsänderungen feuchter Luft. – Meteorol. Z. **1**, 421–431.