Honors Algebra II, Courant Institute, Spring 2022
http://www.math.nyu.edu/faculty/goodman/teaching/HonorsAlgebraII2022/HonorsAlgebraII.html
Check the class forum for corrections and hints

Assignment 10

Due: Thursday, April 28.

From Chapter 15 of the textbook

- 2.1 (an example illustrating the proof that $\mathbb{Q}[\alpha]$ is a field)
- 2.3 *Hint.* The fields $\mathbb{Q}[\alpha] \subset \mathbb{C}$ for different roots α of the same irreducible polynomial are isomorphic (why?)
- 3.1 *Hint.* the title of Section 15.3
- 3.2 The polynomial is chosen to make it easy to check that it's irreducible over $\mathbb{Q}$ (how?).
- 3.6 *Hint.* One way to do this is to use the hint from Exercise 2.3 to show that the a^k are linearly independent for $k = 0, 1, 2, 3$. Another way is to find a related b that is a root of an irreducible $f \in \mathbb{Z}[X]$, where irreducibility is easier to show.
- 4.1 There's a "dumb" way to do this: write $a + b\gamma + c\gamma^2 + d\gamma^3$ in terms of $1, \alpha, \alpha^2$ and find linear relations between a, b, c, d. Probably the author (Michael Artin) had a "smarter" method in mind, but I like dumb.
- 5.2a (not part (b)) This is an answer to **Corollary 15.5.9**.
- 6.1 First see that this is true in examples.
- 7.5 To interpret the result: Every element of $\mathbb{F}_9$ is a root of $X^9 - X$. Three of these are in $\mathbb{F}_3$.
- 7.10 Problems with a $*$ are harder and often involve clever tricks that professionals like Artin know because somebody told them. Don't work on this too long if you have better things to do.