Fall 2004 Calculus I, sections 4, 5, 6, Courant Institute of Mathematical Sciences, NYU.

Homework 13, due December 13

Self check (not to hand in, answers are in the back of the book):

Section 7.7: 3, 13, 25, 39, 45, 57.

Section 8.1: Do all problems 1-46 (except 30 and 34). This is great practice for the final.

Section 8.2: 1, 3 (do not integrate by parts), 7, 13 ($\ln^2 = \ln \cdot \ln$), 19.

To hand in:

Section 7.7: 2, 14, 26, 42, 46, 58, 76.

Section 8.2: 4, 6($x^3 = x^2 \cdot x$), 10, 16, 20, 28(Integrating by parts twice ($\cos(2x) \to \frac{1}{2}\sin(2x) \to \frac{-1}{4}\cos(2x)$) gives a formula involving $\int e^{3x}\cos(2x)dx$ that you can solve to find $\int e^{3x}\cos(2x)dx$).

Another problem (to hand in). Create a table of approximate values of the function

$$f(x) = \int_0^x \cos(t^2)dt \ , \qquad (1)$$

for x in the range $0 \leq x \leq 2$. Since there is no formula for $f(x)$, we have to use an approximation. We will use the *Riemann sum* approximation to the integral (see SHE, figure 5.3.2). Choose $h = .2$. Define $t_k = k \cdot h$, so that $t_0 = 0$, $t_1 = .2$, $t_2 = .4$, etc. The rectangle R_k will have base going from t_{k-1} to t_k, so that R_1 runs from 0 to .2, etc. The height of R_k will be $\cos(t_{k-1}^2)$, so that the height of R_1 is $\cos(0^2) = 1$, the height of R_2 is $\cos(t_1^2) = \cos(.2^2)$ (remember to use radians), etc. The area between the curve $\cos(t^2)$ and the x axis is approximately the sum of the areas of the rectangles, remembering to count the rectangles that hang below the axis as negative areas. The approximate area is

$$g(x) = \sum_{t_k \leq t} \text{area}(R_k) \ ,$$

so that $g(0) = 0$,

$$g(t_1) = \text{area}(R_1) = h \cdot \cos(t_1^2) = .2 \cdot \cos(.04) \ ,$$

$g(t_2) = \text{area}(R_1) + \text{area}(R_2)$, etc. Make a table of the values $g(t_0)$, $g(t_1)$, $g(t_2)$, and so on up to $g(t_{10})$. Plot these values to get a picture of the graph of $f(x)$ for x in the range $0 \leq x \leq 2$. You need to use a calculator for this. The purpose of this problem is to make concrete the idea that a formula like (1) really defines a function of x. This seemed to confuse many people in the chain rule problems like $\frac{d}{ds}\int_0^{\sqrt{s}} \cos(t^2)dt$.