Fall 2004 Calculus I, sections 4, 5, 6, Courant Institute of Mathematical Sciences, NYU.

Homework 6, due October 18

Self check (not to hand in, answers are in the back of the book):

Section 4.2: 11.
Section 4.4: 5, 15.
Section 4.5: 1, 13.
Section 4.6: 5, 33.
Section 4.8: 3, 21, 39, 53.

To hand in:

Section 4.3: 10, 24.
Section 4.4: 6, 16.
Section 4.5: 2, 14 (trick question).
Section 4.6: 8, 34.
Section 4.8: 4, 22, 38, 54.

More problems (to hand in)

1. Suppose $f(t)$ is a function with $f(0) = 0$ and $f'(t) = \cos\left(\pi t^2\right)$. Sketch a graph of f for t between 0 and 2 by following the steps below. Make the distance between $t = 0$ and $t = 2$ along the t-axis at least four inches long.

   (a) Make an explicit list of critical points and inflection points.

   (b) Estimate how much f changes from $t = 0$ to $t = 2$ in the following manner. The change in f over a time interval is the length of the time interval times the average speed. For this problem, the speed is given by the derivative function $\cos\left(\pi t^2\right)$. The average speed will then be the average of this function.

   Draw a good graph of $f'(t)$ and use the graph to estimate averages by eye. For example, between $t = 0$ and the first critical point $t = \frac{1}{\sqrt{2}}$, $f'(t)$ is a bit above $\frac{1}{2}$ for more than half of the time. This means that the average speed between $t = 0$ and $t = \frac{1}{\sqrt{2}}$ is a bit more than $\frac{1}{2}$, so the change in f over this range will be a bit more than $\frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{1}{2\sqrt{2}}$. This suggests that $f\left(\frac{1}{2}\right) - f(0) = f\left(\frac{1}{2}\right)$ is a little more than $\frac{1}{2\sqrt{2}}$.

   Continue in this manner from critical point to critical point until you reach $t = 2$.

   Note #1: Another way to estimate the values of f at the critical points is to put in the known slopes and draw a graph with those slopes.

   Note #2: For those of you hoping to apply prior knowledge of integration to this problem, don't. $\cos\left(\pi t^2\right)$ does *not* have a closed-form integral.