Assignment 3.

Given November 13, due November 27.

Objective: To practice the basics of Monte-Carlo and statistical error bounds

Step 1: Generate iid standard normal random variables, X_j, $j = 1, 2, \cdots$, using the Box Muller method. Verify their Gaussian distribution by plotting as a histogram:

$$N(k,n) = \#\left\{X_j \in (x_k - \Delta x/2, x_k + \Delta x/2) \ , \ \ 1 \le j \le n\right\} \ \ ,$$

and comparing this to

$$e(k,n) = E\left[N(k,n)\right] \approx n\Delta x \frac{1}{\sqrt{2\pi}} e^{-x_k^2/2} \ \ .$$

For large n and Δx small but not too small, $H(k,n)$ and $e(k,n)$ should be close. In this context, the intervals $(x_k - \Delta x/2, x_k + \Delta x/2)$ are called bins.

Step 2: Estimate

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^6 e^{-x^2/2} dx = E\left[X^6\right] = 15$$

using Monte-Carlo and iid standard normals from Box Muller. Choose a sample size so that the estimate is within 1% of 15 with 95% confidence.

Step 3: An insurance company is insuring against five natural disasters. The probability of distaster D_i is $p_i = \Pr(D_i)$. The disasters are independent events. If D_i happens, the company must pay S_i, which is a Gaussian random variable with mean m_i and standard deviation σ_i.

(a) Make a histogram of the probabilities of various payout amounts.

(b) Compute the probability of a payout of $\$10^9$ or more to within 5%.

i	1	2	3	4	5
m_i ($\times\$10^6$)	251	194	620	432	57
σ_i ($\times\$10^6$)	68	95	119	120	10
p_i	.1	.3	.2	.06	.8