Assignment 5.

Given December 11, due end of term.

Objective: To do sensitivity analysis within Monte-Carlo

A risk neutral player has a model for the mean increase in value of a stock that implies that high priced stocks increase in value more slowly than cheap stocks. Hew stochastic differential equation is

$$dS = r(S)Sdt + \sigma S dZ \quad , \tag{1}$$

where

$$r(S) = r_0 + \Delta r e^{-S/g} \quad .$$

The Black-Scholes equation for this is

$$\partial_t f + r(S)S\partial_S f + \frac{\sigma^2 S^2}{2}\partial_S^2 f = r(S)f \quad .$$

She wants to value a put whose parameters are as given in problem 2. The new parameters are $g = \$20$ (the effect takes hold approximately at \$20 stock price) and $\Delta r = .06$ (the premium in growth rate for cheap stocks is at most 6%). Use Forward Euler to compute $f(S = 20, t = 0)$ for a six month expiration. Also compute

$$\frac{\partial f}{\partial g} \quad \text{and} \quad \frac{\partial f}{\partial \sigma} \quad .$$

Here f is short for $f(S = 20, t = 0)$. Use the multihistogram method to compute f as a function of σ for as large a range of σ values as possible. Comment on the admissible range of σ values as a function of Δt. Make sure to put error bars on all Monte Carlo estimates, especially plots produced by the multihistogram method.