together with its "initial data" (at the expiration time). Compare estimates of this sensitivity computed by solving this problem to estimates gotten by solving the original problem and then using divided differences. Prepare a few well chosen graphs to illustrate the answer and the performance of the method. Describe how you tested your code to make sure it given the right answer.

Assignment 3.

Given October 15, due October 22.

Objective: To explore the forward Euler for a multi factor model.

The Bergman Goodman (BG) pricing model for a European put on a non dividend paying bank stock is a two factor model that takes into account stochastic fluctuations in the stock price and the "risk free" short term interest rate. The interest rate, $r(t)$, is governed by a form of the Hull and White model

$$dr = a(r_0 - r)dt + r\sigma_r dZ_r \quad , \tag{1}$$

where r_0 is the natural interest rate, a is a time constant governing the rate of return of the interest rate to its natural value, $Z_r(t)$ is a Wiener process that drives fluctuations in the interest rate, and σ_r is the volatility of the interest rate. The stock price, $S(t)$, is governed by

$$dS = rSdt + S\sigma_S dZ_S + \frac{mS}{r}dr \quad , \tag{2}$$

where $Z_S(t)$ is a Wiener process independent of Z_r. The only new thing is the last term, which quantifies our assumption that any change in interest rates has an immediate effect on the stock price. From the theory of stochastic processes, we know that the expected discounted value of the put, $f(S, r, t)$, satisfies the BG equation:

$$\begin{aligned} \partial_t f &+ \left(rS + \frac{amS(r_0 - r)}{r} \right) \partial_S f + a(r_0 - r)\partial_r f \\ &+ \frac{1}{2}\left(S^2 \left(\sigma_S^2 + m^2\sigma_r^2 \right) \partial_S^2 f + 2mrS\sigma_r^2 \partial_r \partial_S f + r^2\sigma_r^2 \partial_r^2 f \right) = rf \quad . \end{aligned} \tag{3}$$

The expiration and possible exercise time of the put is T. At this time, the value of the put is known to be

$$f(S, r, T) = \max(K - S, 0) \quad ,$$

where K is the strike price. The problem parameters for this particular stock are (with time measured in years)

$$r_0 = .05 \quad , \quad a = 1 \quad , \quad \sigma_S = .5 \quad , \quad \sigma_r = .4 \quad , \quad m = -2 \quad , \quad T = .5 \quad , \quad K = 20 \quad .$$

Use the forward Euler method with constant $\Delta S/S$ and $\Delta r/r$ on a computational domain $S_{\min} \le S \le S_{\max}$, $r_{\min} \le r \le r_{\max}$ with suitable boundary values on the boundary. Do a convergence study to determine the order of accuracy of your method. Choose a time and space step so that the error in $f(S = 20, r = .04, t = 0)$ is less than 1%. To determine the sensitivity of this price to changes in the volatility, find the equation satisfied by

$$\frac{\partial f}{\partial \sigma_S}$$