# Assignment 1.

Given September 3, due September 17.

**Objective:** To explore dynamic programming.

This is a qualitative model. It is unrealistic in several ways in order to be as simple as possible. We suppose that a floating interest rate fluctuates between 1% and 10% (annualized). Now it is 5%. Each week it may move up or down by .2%, except that it is blocked from moving outside its range. It moves in either direction with probability 1/3 and remains unchanged with this probability. All movements are independent. This interest rate is $R(t)$. A loan of \$1 is to be repaid in 250 weeks. Each week, we add some interest and make a payment. If our interest rate at week $t$ is $r(t)$, and the outstanding balance is $B(t)$, then, at week $t$ we first add interest, replacing $B(t)$ by $B'(t) = (1 + r(t)/52) * B(t)$. Next we make a prorated payment, $P(t) = B'(t)/(250 - t)$. This leaves $B(t + 1) = B'(t) - P(t)$. Our interest rate is determined as follows. At week 0, $r(t) = R(t) = 5\%$. However, $r(t)$ remains fixed until we "refinance". Whenever we refinence, $r(t)$ is replaced by $R(t)$. We are allowed to refinance up to four times during this 250 week period. We want to do so so to minimize our total expected payments, $\mathbf{E}[P(0) + P(1) + \cdots + P(249)]$. Set this up as a dynamic programming problem. Identify the state space. Write a computer program to solve it by dynamic programming. Use interpolation to estimate unknown values of the cost to go function from previously computed values.