Assignment 5.

Given November 19, due December 10.

Objective: To explore sensitivity analysis for stochastic differential equations.

We have an ad-hoc interest rate model

$$dX = \mu X dt + \left(aX + b\sqrt{X} \right) dZ \quad , \qquad X(0) = 1 \quad .$$

In a contract, a company agrees to pay one dollar a year from now as long as the interest rate is less than M at all times before then. Otherwise, the company will pay nothing. We formulate this mathematically by saying that there is a "characteristic function"[1] which is zero or one depending on whether the the barrier is reached before time $T = 1$. That is

$$\chi = \begin{cases} 0 & \text{if } X(t) \geq M \text{ for some } t \leq 1, \\ 1 & \text{if } X(t) < M \text{ for all } t \leq 1. \end{cases}$$

The present value of the payout is

$$F = \mathbf{E}\left[\exp\left(- \int_0^1 X(t) dt \right) \chi \right] \quad .$$

The parameter values are $\mu = 1$, $a = 1$, $b = 0$, $M = 3$. Use the forward Euler method to solve the stochastic differential equation and estimate F to 2 decimal places.

This assignment asks you to try several different methods for estimating the sensitivities

$$\frac{\partial F}{\partial \mu} \; , \quad \frac{\partial F}{\partial b} \; , \quad \frac{\partial F}{\partial M} \quad .$$

Some of the methods may not be applicable to each of these problems. If you write a flexible program, you should be able to do the experiments called for fairly quickly, even though there seem to be many of them. Please to not spend an unreasonable amount of time on this. The methods for estimating sensitivity are

- Dumb finite differencing, with one sided or centered differencing
- Finite differences using the "same paths", again with one sided or centered differences.
- Analytic differentiation of the stochastic differential equation with respect to the parameter.
- The score function method.

Comment on which methods are applicable to which sensitivities. Try to find a reasonable step size for the finite difference methods. Comment on the accuracy (effectiveness) of the methods for the various sensitivities.

[1] Not the Fourier transform, which is sometimes called characteristic function by probabilists.