Derivative Securities, Courant Institute, Fall 2008
http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec08/index.html
Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment.

Assignment 1, due September 10

Corrections: (Sept. 5) problem 2c and 3a.

1. Suppose the present exchange rate between the US Dollar and the Euro is .7 Euros per Dollar, the price of a 180 day US Treasury bill is $80 per $100 face value, and the price of the analogous Euro instrument is 90 Euros per 100 Euro face value (not real numbers).
   (a) What was the theoretical 180 day forward exchange rate?
   (b) Suppose the market 180 day forward exchange rate was .65 Euro per Dollar. Describe the risk free strategy for making money in this market.
2. Let $B(t,T)$ be the cost at time t of a risk free dollar at time T.
   (a) suppose $B(0,1)$ $B(1,2)$, and $B(0,2)$ are known at time 0. Show that the absence of arbitrage requires that $B(0,1)B(1,2) = B(0,2)$.
   (b) Suppose that $B(1,2)$ is not known at time $t = 0$. What goes wrong with your argument for the formula in part (a)?
   (c) Show that if we know at time $t = 0$ that $B(1,2) \geq m$, then $B(0,1)\, m \leq B(0,2)$.
   (d) Even when $B(1,2)$ is not known, it might be possible to enter today into a contract to pay X (actually, $X(0,1,2)$) at time $t = 1$ to receive 1 at time $T = 2$. This is the *forward price*). In the real world, we probably will learn at time $t = 1$ that $X \neq B(1,2)$. Question: is the assumption of part (a) the same as assuming that this does not happen, i.e., that we know today that $X = B(1,2)$?
3. The present price of a stock is 50. The market price of a European call with strike 47.5 and expiration in 180 days is 4.375. The cost of a risk free dollar 180 days hence is $B(0,180) = .98$.
   (a) Show that a put price of 1.450 violates put/call parity.
   (b) Describe how to make a profit with no risk from these prices.
4. An investor holds a European call with strike K_c and maturity T on a non-dividend-paying asset whose current price is S_0. Suppose the investor can write (i.e. sell) a put with any strike price K_p, can write a forward with any delivery price, K_f, and can borrow or lend any amount, X, at the risk-free rate, r. What are the conditions on K_p, K_f, and X that make this combination of positions a constructive sale (see Kohn's notes for the definition of constructive sale).
5. Suppose there are n currencies and a dealer has a table of exchange rates, P_{jk}, for $j \rightarrow k$. The dealer will give P_{jk} units of currency k for one unit of currency j. Assume that $P_{jk} = 1/P_{kj}$ for every pair $j \neq k$ (and that $P_{kk} = 1$, for every k). A three currency arbitrage is a triple of currencies, j,k,l, so that $P_{jl} \neq P_{jk}P_{kl}$.
   (a) Show that a three currency arbitrage as described above is actually an arbitrage opportunity.
   (b) Show that if the table has no three currency arbitrage opportunities then it has no arbitrage opportunities involving more than three currencies.
   (c) Show that if there is no arbitrage opportunity in the table, then all the exchange rates are determined by the exchange rates from currency $j = 1$. That is, if we know the rates P_{1k} for all k, then we know the rates p_{jk} for all j and k.