

Derivative Securities, Courant Institute, Fall 2008

<http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec08/index.html>

Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment.

Assignment 3, due October 8

Corrections: (none yet)

1. The payout of a *power option* at time T is S_T^n . Find the Black Scholes price of a power option.
2. A *digital option* has payout equal to one if $S_T \in [a, b]$ and payout equal to zero otherwise.
 - (a) What is the Black Scholes price of a digital option?
 - (b) Find a formula for Δ for such an option. Show that $\Delta \rightarrow \infty$ when the option is close to expiration and the spot price is close to a or b .
3. This is the beginning of a series of computational assignments that require you to use Microsoft Excel and (eventually) Visual Studio. You should start figuring out how you will access these products, either by buying them or getting access to a computer that has them. This week involves only Excel. There is a help document for this assignment that guides you through its computer aspects. You will construct spreadsheets but not hand them in. Instead, you will hand in printouts of that document how you did some of the work. Your spreadsheets need not be identical to the ones in the help document, but they should have more or less the same functionality. The ones in the help document are organized to be easy to read by someone taking Derivative Securities (e.g. intermediate calculations displayed) and easy to manipulate to do the various parts of the assignment.
 - (a) In Excel, implement the Black Scholes formula that prices vanilla European puts and calls. Also implement the formulas for Δ , Γ , and Λ (called “Vega” for some reason). Use your spreadsheet to find the price of a put and a call that expire in half a year each with strike price 40. Assume the present spot price is 42, the risk free rate is 10%/year, and the volatility is 20%/year. Check that your results satisfy put/call parity. (Do this in the spreadsheet. Note that the help files do not display the spreadsheet with this check.) Experiment with different values of the parameter and see that put/call parity always works. Try to find extreme values for which put/call parity starts to fail in the spreadsheet, possibly because of inaccurate evaluation of the cumulative normal, $\mathcal{N}(x)$. Hand in a printout of the spreadsheet that verifies put/call parity.

- (b) Verify that the Greeks are programmed correctly by checking that the formulas agree with finite difference approximations. For example, we should have (P is the put price. C is the call price.)

$$\begin{aligned}\Delta_p &= \partial_{s_0} P(s_0, r, \sigma, T, K) \\ &\approx \frac{P(s_0 + \delta s_0, r, \sigma, T, K) - P(s_0, r, \sigma, T, K)}{\delta s_0}.\end{aligned}\quad (1)$$

To verify Γ , use the centered second difference approximation

$$\partial_x^2 f(x) \approx \frac{f(x + \delta x) - 2f(x) + f(x - \delta x)}{\delta x^2}.$$

Of course, you should apply this to the variable s_0 in P and C . Use the one sided difference to verify Λ . Hand in a printout of the spreadsheet that verifies these derivatives. You have to choose δs_0 and $\delta \sigma$ carefully. If they are too big, the finite difference approximations are not accurate. If they are too small, the calculations of P and C are not accurate enough. We are not asking for the optimal δs_0 , but you may have to experiment to find a good one. Use the parameter values given in part a.

- (c) The *centered* finite difference approximation is usually more accurate than the one sided one. That is

$$\partial_x f(x) \approx \frac{f(x + \delta x) - f(x - \delta x)}{2\delta x}$$

is usually a better approximation than

$$\partial_x f(x) \approx \frac{f(x + \delta x) - f(x)}{\delta x}$$

Check whether this is the case for finite difference estimation of Δ_p .

- (d) Expand your spreadsheet so that it can handle a small portfolio of put and call positions on three or four distinct strikes. Calculate the total value (the weighted sum of the individual option values weighted by the number of options in the portfolio) of the portfolio and its net sensitivities (also weighted sums). Use this to find a butterfly spread that expires in 6 months (same market parameters) that has total Δ close to zero. What is the Λ for this? How sensitive is this butterfly spread to uncertainties in the volatility parameter? Hand in a spreadsheet printout with the butterfly portfolio you found.