Derivative Securities, Courant Institute, Fall 2008

http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec08/index.html

Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment.

Assignment 5, due October 15

Corrections: (none yet)

1. This problem asks you to use Excel to plot. We will work with a vanilla put with parameters $T = .5$, $\sigma = .3$, $r = .05$, $K = 50$. The help file associated with this assignment has hints on how to do this.

   (a) On a single chart, put plots of the put price, Δ, and Γ, for s in the range $30 \leq s \leq 80$.

   (b) Repeat part (a) with $T = .2$. Describe the differences between the plots, in particular the differences in Γ. Why do short dated options have more Γ?

2. There are simple approximate formulas for put and call prices when the options are deep in the money or deep out of the money.

   (a) Argue that a deep in the money option is almost certain to be exercised and therefore that it is similar to a forward contract. Use this to give approximate formulas for put and call prices that apply when the spot price is deep in the money.

   (b) Give a derivation of the formulas from part (a) using the fact that $N(x) \to 1$ as $x \to \infty$ and $N(x) \to 0$ as $x \to -\infty$.

   (c) Derive an approximate formula for $N(x)$ that applies when x is a large positive or negative number that is more accurate than simply $N \approx 0$ or $N \approx 1$. For large positive x, use

   $$1 - N(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-z^2/2}\, dz \ .$$

   Use the change of variable $z = x + y$ and the approximation $z^2 \approx x^2 + 2xy$. Show that this approximation is valid where the integrand $e^{-z^2/2}$ is significantly different from zero, in the case of large positive x.

   (d) Use the result of part (c) to get approximate formulas for put and call prices that hold when the spot price is deep out of the money.

   (e) Make a plot using the parameters and price range of Problem 1a that also includes the approximate formulas from parts (a) and (d) here. This plot should have the put price and approximations only, not Greeks.

3. Suppose we have a market put price $\overline{P}$, and know the values of s, r, K, and T. The value of σ so that $P(s, \sigma, r, K, T) = \overline{P}$ is called the *implied volatility* of that option price.

(a) Show that if an option price has an implied volatility, that implied vol is unique. Hint: What is the sign of Λ?

(b) Find an approximate formula for the implied vol of a put that holds when the put is deep out of the money. Use the result of Problem 2d.

(c) Use the solver in Excel to calculate the implied vols of a put with parameters $s = 42$, $r = 10\%$, $K = 40$, and $T = .5$ if the market put price is .8 and .9.