Derivative Securities, Courant Institute, Fall 2008

http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec08/index.html

Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment.

Assignment 7, due October 29

Corrections: (none yet)

This assignment asks you compute put and call prices using a C++ program. You may write the program from scratch, but it probably will be easier to download the code posted with the assignment in the form of the files `main.cpp`, `TimeStep.cpp`, `FinalValues.cpp`, and the header file `prototypes.h`. If you're using the Microsoft system Visual Studio or the Apple system Xcode (I've tried it on both), you can create a project with one file for each procedure and a header file, build the project and run it in the debugger. It should work with any C++ compiler, including the gnu compiler on a linux box. It will create a file `outcake.csv`, which contains the computed results together with some information about the run. You can see this output by opening `outcake.csv` with Excel. On a Windows (Microsoft) box or a Mac (OS X), you just double click on the file icon and it opens it automatically with Excel. The last two rows are the computed solution. You should plot them using a scatterplot, as you did in Assignment 5. The result should look like the `OutOfTheBox.gif` file posted with the assignment. Warning: you probably will have to change the output file name. The one in the out of the box code is for a Windows laptop I had the misfortune of using when I created it. I think a Mac or Linux box will use the correct file path, rather than the Microsoft version with double slashes between the names of the directories. Use Excel to make all the plots asked for in this assignment.

The `.csv` format ("csv" stands for *comma separated values*) us just a bunch of numbers separated by commas. Excel puts them in consecutive cells in the spread sheet. Look at the out of the box code to see how to create a `.csv` file. It should be clear what to do if you want to transfer more or different information about the run to Excel.

Read the supplementary notes carefully before starting the work below.

1. Modify the out of the box program to price a the European put with parameters $K = 100$, $T = 1$, $r = 10\%$ (a pretty big value), $\sigma = 30\%$. Compute and plot the solution for $30 \leq s \leq 150$. Note, since the grid points are uniformly spaced in log space, you will have to write to the `.csv` file the s values corresponding to the uniformly spaced x values used in the computation. It will be clear in the scatterplot that the s values are not uniformly spaced. This actually is a generally good thing about using the x variable rather than the s variable in computing. Experiment with various values of δx until you are satisfied that making δx smaller does not noticeably change the plot.

Check that the results are correct by recomputing the prices using the Black Scholes formula that should be in spreadsheets from earlier assignments. When δx is small enough, the curves should lie on top of each other.

2. Repeat Problem 1, but now for an American style put with the same parameters. Plot the European and American prices on the same plot to see how much *American premium* there is. Pay attention to the price curve near the early exercise point, which probably is hard to find if you don't mark it explicitly in the plot. Comment on whether you are able to observe the *smooth pasting* condition. This is the condition that $\Delta = -1$ on both sides of the early exercise point. Of course $\Delta = -1$ in the early exercise region (why?). Also $0 > \Delta > -1$ in the hold region. But $\Delta \to -1$ at the early exercise boundary, so that the price curve in the hold region pastes smoothly onto the intrinsic value curve on the early exercise side. It will help to plot the intrinsic value curve as well.

3. Make a plot of $b(t)$ as a function of t for $0 \leq t \leq T = 1$ for the American put of Problem 2. This part of the assignment will tax the computer more than the earlier parts because you have to take δx very small in order to get something you would want to show to your boss. Comment on the following aspects of the plot and give explanations if possible

   (a) What is $\lim_{t \to T} b(t)$? Where is the early exercise boundary very near the expiration time?

   (b) How fast is the early exercise boundary moving very close to expiration? What is $\lim_{t \to T} \partial_t b(t)$?

   (c) (*Only attempt if you have lots of free time and are very motivated*) What is the behavior of the American premium as t approaches T? How do you reconcile this with the result of part a?

4. (*Describe the algorithm but don't code it or run it.*) What would you do to compute the Black Scholes price of a Bermudian style American put with expiration T and early exercise dates $T_1 < T_2 < T$.

5. Show that an American style call option is never exercised early in the Black Scholes theory unless there is a q (cost of carry, or dividend rate). Show that an American style call option may be exercised early in some cases if $q > 0$. Hint for both parts: how does the price of a European style call compare to the intrinsic value (deep in the money for the second part)?