Assignment 8, due November 5

Corrections: (none yet)

1. Attempt to repeat Problem 1 of Homework 7 (valuing a European put using the finite difference method) but with the CFL ratio equal to 2. This is called `DXRATIO` in the code and set in the statement `#define DXRATIO .5`. This does not work; it fails to give accurate estimates of the solution of the PDE. Hand in a plot or two that illustrates what does happen. If your plots look at all like the solution of the PDE from last week, you have done something wrong. Note and comment on the size of the computed results. Explain why this is called *numerical instability*.

2. The supplementary notes for Section 8 have a discussion of the fact that there is a natural direction for time to flow in a PDE, even though there may not be such a natural direction in an ODE. To explore this, try to solve the Black Scholes equation in the wrong time direction. More specifically, try to find a payout function, $V(s)$ so that the option value at time $t - 0$ is $f(s, 0) = (K - s)_+$. As with Problem 1 above, if this works you have done something wrong.

   (a) Formulate this is an *initial value problem*. That is, as the solution $f(s, T)$ of the Black Scholes PDE at time T with initial conditions $f(s, 0) = W(s) = (K - s)_+$.

   (b) Convert this PDE to a PDE with constant coefficients as we did before for the Black Scholes PDE using the log variable.

   (c) Create a finite difference marching method for this PDE that computes the approximate values $F_{j,k+1}$ from the approximate values F_{jk}. This differs from the finite difference equations we used in Assignment 7 in that it uses

   $$\partial_t f(x_j, t_k) \approx \frac{f(x_j, t_{k+1}) - f(x_j, t_k)}{\delta t} \approx \frac{F_{j,k+1} - F_{jk}}{\delta t}$$

   to get a formula of the form

   $$F_{j,k+1} = aF_{j-1,k} + bF_{jk} + cF_{j+1,k} .$$

   What are a, b, and c, in terms of r, σ, δx, and $\lambda = \sigma^2 \delta t / \delta x^2$?

(d) Adapt the code from Assignment 7 to implement this – to use the difference equations from part c to move the initial data from part a up to time $T = 1$. Make one or two plots that illustrate what happens. Unlike Problem 1, this misbehavior is required by mathematical law and cannot be cured by changing λ or using a different solution method. Try a few values and see.

3. Download the file `Homework8.pdf` and build the spreadsheet described there. Hand in the action items.