

Supplementary notes and comments, Section 1

The reason for options to exist. Why are there large options markets? The original reason is as explained in the notes. People want to eliminate market risks by locking in prices in advance of the actual transaction. In this sense, options are similar to insurance. Someone who does not want to assume certain risks pays someone else to assume them. Options markets allow the risk to be offloaded to a large number of investors each of whom may be willing to accept a small fraction of the total risk. An airline may need \$10⁹ in currency options, which can be distributed in the market with a large number of counter-parties.

Once a market exists for anything, speculators may trade on their private views of future market moves. This is more than a mechanism for day traders to lose their savings to market professionals. Active trading means *liquidity*, so that there always are reliable price quotes and available counter-parties.

Many investors use options as a way to bet on the underlying asset. If a stock price goes up, call options on the stock should go up as well. The pricing theory of Black and Scholes implies that betting on the stock by buying options is exactly equivalent to betting by buying the stock directly. This is part of the Black Scholes partial differential equation the *Black Scholes equation*, for short. Of course, the assumptions behind the Black Scholes equation are not satisfied in real markets, so there may be reasons to prefer to use options rather than only the underlying security.

Some of these reasons are trading and accounting rules. For example, if you want to bet that a stock will decline, it may be easier and cheaper for a private investor or a constrained mutual fund to purchase put options than to sell the stock short. For example, China forbids individual Chinese investors to sell short in Chinese markets. Many mutual funds also are barred from short sales. Even when short selling is allowed, there may be substantial carrying costs associated with borrowing the stock. Investment banks have easier and cheaper access to short sales, but accounting rules may treat the risk of option ownership differently from the risk of stock ownership, which impacts capital requirements banks face. Modernization of accounting rules is making this less of an issue.

Another reason is transaction costs and liquidity risks. The Black Scholes equation is based on a trading strategy that relies on a large number of very small trades (an infinite number of infinitely small trades). It also relies on the price to buy, the *ask* price, being equal to the price to sell, the *bid* price. In real markets, the bid price is slightly less than the ask price, and prices do not move continuously. These are some of the factors that break the strict relation between stock and option prices.

Finally, option prices are *nonlinear* functions of underlying stock prices. Therefore, one can make nonlinear bets on the market. For example, one can bet that the market will make a large move without knowing the direction of that move.

Arbitrage and statistical arbitrage. The standard definition has *arbitrage* being a trade or trading strategy that is guaranteed not to lose money and with positive probability make money. On the other hand, *statistical arbitrage* is a collection of trades that is likely to make money and hopefully unlikely to lose money. In real markets, it is almost impossible to find a true arbitrage in the strict sense. Still, the idea of looking at the implications of complex dynamic (time dependent) trading strategies is very important in modern finance. The distinction between statistical arbitrage and other hedge fund activities is shrinking.

Statistical arbitrage also has a more specific technical meaning as a way to profit from mergers.

Whose hedging? The arbitrage arguments are sometimes used to identify arbitrage opportunities as above. They also are used to predict the relationship between a derivative price and the price of the underlying asset. If the spot price of oil goes up, what will happen to oil futures? These relations are derived using hedging arguments that most people would be unable to actually do. For example, most people cannot buy and store lots of crude oil as would be required to profit from the forward price being too high. Instead, the idea is that *someone* can do it. This should be enough to keep the forward price in line with the spot price.