Derivative Securities, Courant Institute, Fall 2009

http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec09/index.html

Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment.

Practice for the final exam

Corrections: (none yet)

Important:

- The final exam is on Wednesday, December 23, from 7:10 to 9 pm in room 408 of the Silver Center. Please arrive a few minutes early because access to the Silver center is slow.
- You only may bring one piece of paper with whatever information you want to put on it. You must be able to read this without magnifying lenses or electronics. No other materials, electronics, or books are allowed.
- You may not use and will not need a calculator.
- The questions below are examples of the kind of questions that will be on the actual final. The questions on the final will be different, though some may be similar.
- I will have special office hours from 5 to 8 on Monday and Tuesday, December 14 and 15.
- The actual exam will be much shorter than this.

The Questions

Part 1, True/False. *In each case, indicate whether the statement is true or false. Please explain your answer in a few words. You may receive no credit for a correct answer with no explanation.*

1. The Δ of a put option is negative.
2. The Γ of a call option is positive.
3. The Δ of an American style option is continuous at the early exercise boundary.
4. The Γ of an American style option is continuous at the early exercise boundary.
5. The price of a 30 year zero coupon bond is more sensitive to fluctuations in interest rates (parallel shifts in the yield curve) than the price of a 30 year coupon bearing bond. (*note: This is theoretical. Almost all 30 year bonds have coupons.*)

6. It is possible to determine the 30 year risk free yield from the price of a 30 year coupon bearing Treasury bond.

7. In the Black Scholes theory, an American style call option on a dividend paying stock with volatility σ is more valuable the same option (same expiration and strike and features) on a stock with volatility σ that does not pay dividends.

8. If the real world stock process is $dS = \mu S dt + \sigma S dW$ and $\mu > r$ (the risk free rate), then the discounted expected return of a European call option is larger than its Black Scholes price. Assume discounting by a fixed e^{-rt}.

9. If we make a bond callable but do not change any other features, it becomes more expensive.

10. We often model an index as a lognormal process. The index is the weighted sum of the equities in the index (for an equity index). The sum of two (or any finite number) of lognormal processes is not lognormal.

Part 2, multiple choice

1. An option that may be exercised on certain specified dates is called

   (a) Asian

   (b) Azoran

   (c) Bermudan

   (d) Lookback

2. An option is *path dependent* if its payout may not be determined from the value of the underlier at the expiration time only. Which of the following is not path dependent:

   (a) American style put

   (b) European style put

   (c) Asian style put

   (d) Lookback

3. Suppose we want to pursue a Δ hedging strategy to hedge a vanilla equity option. Suppose that we want to take into account the fact that future interest rates will not be what they are today and are uncertain. Since we may have to borrow or lend to do the Δ hedge, we can enter a sequence of forward rate agreements to guarantee the borrowing/lenging costs/returns. Which of the following is a flaw in this reasoning?

   (a) The amounts to be borrowed or lent are not known today.

   (b) The costs of borrowing or lending at time t are not known today.

   (c) The costs of forward rate agreements are not known today.

(d) There may be a correlation between the underlying equity price and fluctuations in interest rates.

4. We have cash today that we wish to place in a credit risk free instrument until time T. Two possibilities are (i) to buy zero coupon long dated Treasury bonds today that mature at time T, or (ii) to buy Treasury zero coupon Treasuries that mature at time $t < T$, or (ii) to buy short term zero coupon Treasury bills that mature at time $t = T/n$ and keep re-investing in t/n T-bills until time T. Which of the following is true? (*Note: Treasury Bonds all have coupon payments. However, it is possible to buy Treasury "strips" that have the same maturity but no coupon payments. Look it up in Hull.*)

   (a) Finance theory suggests that the expected return on the two strategies is the same.

   (b) Finance theory suggests that the return on the the long dated bond is higher than the expected return on the T-bill strategy.

   (c) Finance theory suggests that the return on the the long dated bond is lower than the expected return on the T-bill strategy.

   (d) Which of the strategies has a greater expected return depends on the yield curve today.

5. The graph in Figure 1 is most likely to represent the graph of what quantity, as a function of the spot price today?

   (a) The price of a vanilla call option.

   (b) The Delta of a vanilla call option that expires very soon.

   (c) The Delta of a vanilla call option that is far from expiration

   (d) The Gamma of a vanilla call option that is far from expiration

   (e) The Gamma of a vanilla call option that is close to expiration

6. A simple portfolio of options that pays a positive return if the underlier makes a small move and has a negative return (when you take into account the costs of the options) if the underlier has a large move in either direction is called

   (a) A butterfly spread

   (b) A bull spread

   (c) A bear spread

   (d) A yield spread

7. Which of the following is a valid argument for using the binomial tree method to price European style options instead of the continuous time lognormal model?

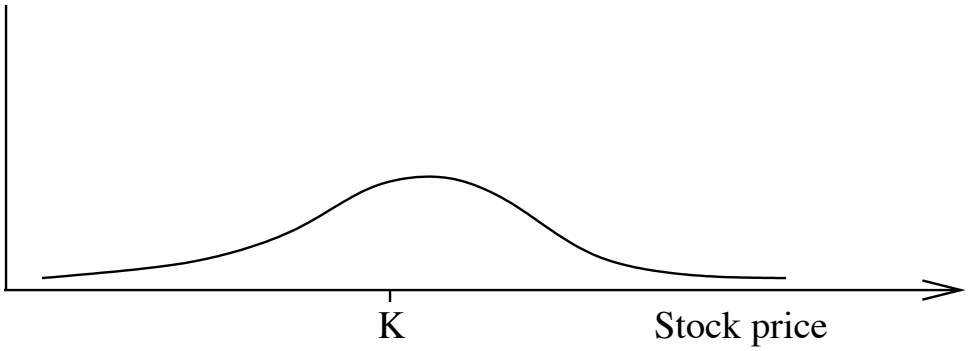

Figure 1: Figure for Part 2, 5. TeX is not good at figure placement.

(a) The formula for the price is simpler in the binomial tree model than in the continuous time model

(b) The binomial model is a more accurate representation of the true dynamics of asset prices than the lognormal continuous time model

(c) The price delivered by the binomial tree model depends on fewer unknown parameters than the lognormal continuous time model

(d) The reasoning in the binomial tree model is more elementary.

8. Where is the Δ of an up and out call call the largest in magnitude (positive or negative). Assume the knockout price is well above the strike price.

(a) Near the strike and close to expiration.

(b) Deep out of the money.

(c) Near the knockout boundary.

(d) Halfway between the knockout boundary and the strike, but close to expiration, where $\Delta \approx 1$.

9. Which is a valid reason not to use a binomial tree to price an option when the local vol is a function of the stock price?

(a) The tree may not be recombining.

(b) It is impossible to hedge an option if the vol can change

(c) Monte Carlo is always more accurate and faster than a deterministic calculation

(d) The binomial tree process is not a valid model of the actual price process if the vol is not constant.

Part 3, Full answer

1. Suppose $dS_t = rS_t dt + \sigma S_t dW_t$ and $S_0 = 1$. Calculate $E[S_T^2]$.

2. Suppose $dr_t = a(\overline{r} - r_t)dt + \sigma dW_t$, with constant parameters a, $\overline{r}$ and σ. Suppose $F_t = e^{A_t + B_t r_t}$, where A_t and B_t are differentiable functions of t. What differential equations must A_t and B_t satisfy in order that F_t should be a martingale?

3. A company announcement will happen ten minutes from now. The possible announcements are `G` or `B`, for good or bad news. `G` means that the default probability on its outstanding and actively traded one year zero coupon bond becomes zero. Just before the announcement, the bond has an implied default probability of 5%. If the announcement is `G`, the stock will immediately jump from S to $1.1 \cdot S$. If the news is `B`, it will jump from S to $.8 \cdot S$. Calculate the new implied default probability the bond will get if the news is `B`. If you need to know, the one year risk free yield is 1%. Assume zero recovery on default. Hint: Let $R_{\mathtt{B}}$ be the implied default probability if the news is `B`. Call the present implied default probability $R_{\mathtt{P}}$. A binomial tree argument allows you to calculate $R_{\mathtt{P}}$ in terms of $R_{\mathtt{P}}$ and the other data. Take that formula and solve for R_B in terms of $R_{\mathtt{P}}$. (Ans = 15%)

4. An *annuity* is a defined income stream purchased at time 0. It is much like a coupon bond, but there is no principal payment at maturity. Consider an annuity that pays a continuous income stream with payment dt in time dt starting at $t = 0$ and ending at $t = T$.

   (a) Find an equation that can be solved to find the yield to maturity of such an annuity, assuming that the price today is L.

   (b) Show that this equation has a unique solution, which is positive, for all L values that make sense. Show that the equation does not have a positive solution if L does not make sense. What is the range of L values that makes sense?

   (c) Calculate the duration of the annuity (in the sense of fixed income instruments) for values of L that make sense. (Ans: $\frac{1}{r} - \frac{T}{e^{rT}-1}$) Show that this answer makes sense in the limits $r \to 0$ and $r \to \infty$.

5. Let r_t be the short interest rate and let $r_B > r_0$ be given. Define τ to be the first t value with $r_t = r_B$. This τ is the *first hitting time*, and r_B is the *barrier* or *trigger* level. Consider a "short rate protection" option that pays one unit at time τ as long as $\tau \leq T$. The hitting time, τ is an example of a *stopping time*, which is defined as a random time so that "you know it when you see it". The *Doob stopping time theorem* states that if Y_t is a martingale, and if $Z_t = Y_{\min(t,\tau)}$, (Z is the Y process stopped at time τ), then Z_t also is a martingale. Suppose that in the world where the money market is the numeraire, the short rate follows an SDE $dr_t = a(r_t)dt + b(r_t)dW_t$.

(a) Show that the price of this option today is given by

$$E\left[\exp\left(-\int_0^{\min(T,\tau)} r_t dt\right)\right] \tag{1}$$

(b) Complete the sketch a C++ program that could estimate (1) by Monte Carlo. Assume that there is a procedure `double zrand()` that produces an independent standard normal random variable.

```
    double a(double r);   // Returns the drift for a given r value
    double b(double r);   // Returns the noise for a given r value

    double xsample( double T, double r0, double rB, int n) {

//  T is the final time
//  r0 is the starting short rate
//  rB is the barrier level for r
//  n is the number of time steps to use

    double dt = T/n; // The time step for the SDE and integration
    double r = r0;   // The short rate at time t
    double I = 0;    // The integral in the exponential

    zrand( Z, n);
    for ( int tStep = 0; tStep < n; tStep++) {
       if ( r > rB ) break;
       I += YOUR CODE HERE;
       r += YOUR CODE HERE . . . *sqrt(dt}*zrand();
     }
    return exp( -I);
  }

   int main() {

      int      N = 1000000; // Number of Monte Carlo samples
      int      n = 100;     // Number of time steps in a path
      double  r0 = .01;     // The starting short rate, in %/year
      double  rB = .1;      // The barrier
      double   T = 5;       // Expiration time for the option
      double sum = 0;       // The Monte Carlo sum

      for ( int samp = 0; samp < N; samp++)
         sum += xsample( T, r0, rB, n);

      cout << "The Monte Carlo estimate is "
```

```
        << YOUR CODE HERE << endl;

    return 0;
}
```

(c) Define a value function $f(r,t) = E_{r,t}[\cdots]$ and state what PDE it satisfies. Give the boundaries and the boundary and final values for this PDE.

(d) Sketch a finite difference method for solving this PDE.

6. Suppose there is an Excel function `BSP(r, T, S, sigma)` that returns the Black Scholes price of a vanilla European put with those parameters. Write a formula one could use in Excel to estimate Λ for the same option using a finite difference approximation to the derivative.

7. A company wishes to take a loan six monty loan of size L paying six month LIBOR + 1%. It wishes to have the right to renew this loan, always using the the six month LIBOR at the time of renewal, for the next five years (maximum of nine renewalls).

   (a) Describe this loan in terms of standard instruments (bonds, stocks, options) and standard features (put, call, European, American, Bermudan).

   (b) Describe how a price would be determined. What market data would you need. What computations would you do?

8. More to come – have to post now.