Derivative Securities

Class 1
September 9, 2009
Lecture outline

latest correction: Sept 14

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http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec09/index.html

Procedures and policies

- Material:
  - Lecture: some material in lecture only
  - Text: Hull
  - Lecture notes: Bob Kohn and Steve Allen
  - Occasional supplementary notes
  - Suggested outside readings, check class web site

Procedures and policies

- Prerequisites:
  - Multivariate calculus
    - Partial derivatives
    - Multiple integrals
    - Lagrange multipliers
  - Calculus based probability
    - Multivariate normal, central limit theorem
    - Conditional and marginals for multivariate random variables
  - Linear algebra
    - Linear systems, solvability
    - Eigenvalues and eigenvectors for symmetric matrices
  - C++ beginner status
  - Excel sub-beginner status

Procedures and policies

- Assignments:
  - Weekly (approximately)
  - Due the following week
  - Deduct 5%/week for late homework
  - Mathematical/financial exercises
  - Computing in Excel and C++
  - Writing: coherent paragraphs

Procedures and policies

- Assessments:
  - Assignments, separate grades for
    - Technical (traditional) work
    - Computing -- correctness, code quality, output clarity, etc.
    - Writing -- correctness (grammar, neatness, spelling, paragraph structure), effectiveness.
  - Final exam, December 23, 7 - 9 pm!
  - Adaptive weighting, starting with 50% - 50%

Procedures and policies

- Collaboration:
  - May consult other students and sources
  - Please post to and read from class message board
  - No text sharing. Every word or formula you hand in must be typed or written by you.
  - No code sharing. Every byte of code and piece of spreadsheet must be done by you.
  - Violating these policies will not benefit you in the long run.
  - Penalties for cheating begin with grade reduction and end with expulsion.
  - Protect your work and report violations

Procedures and policies

- Communication:
  - Open web site for assignments and most materials
  - Blackboard site for message board, grades, some materials, register or see me.
  - Office hours, Mondays 4-6.
  - Email for non-technical issues: goodman@cims.nyu.edu.
  - Technical questions, comments, etc. on the message board. I check it daily.

Financial markets / asset classes

Underliers

- Equities -- stocks
  - Dividends / dividend uncertainty
  - Asset price uncertainty
- Corporate bonds -- defaultable
- Treasury bonds -- pure interest rate
- Commodities -- energy (oil), metals, foods, real estate
- Currencies
- Bundles and tranches, securitization -- CMO, CDO

Derivatives

- Forward contracts -- over the counter (OTC)
- Futures contracts -- exchange traded, daily settlement
- Exchange traded options -- "plain vanilla", nonlinear payout
- OTC options -- exotics
- Swaps

Financial markets / participants / goals

- Companies and individuals seeking financial services
  - Loans, commodities (buy, sell)
- Companies and individuals with capital to invest
- Companies and individuals seeking to reduce risk
  - Forward and futures contracts
  - Options
  - Insurance
  - Bundling
  - Tranching and “financial engineering”
  - *Hedging*: buying anti-correlated assets to reduce risk
- Speculators & arbitrageurs -- hedge funds, etc.
- Agents for the above -- the financial services industry

Forward contract

- The holder agrees to pay K for a specified asset at time T
  - K = *strike price*
  - T = *delivery date*, *expiration* date, *settlement* date, *expiry*, …
  - The holder is *long* the contract
  - The *counterparty* is *short* the contract
  - If you have N contracts, $N > 0$ is long, $N < 0$ is short
  - Contracts may be *settled* in cash or by actual *delivery*
  - Cash settlement: holder pays the counterparty $K - S_T$ at time T
  - S_T = *spot price*
- Over the counter, not in exchanges

Futures contract

- Exchange traded
  - Standardized terms
  - Exchange assumes counterparty risk
  - Exchange mandated margin requirements
  - Published prices
  - Liquidity: ability to buy and sell at or near published prices
  - Daily settlement

- A mechanism for speculating on assets that are hard to buy directly
  - Commodities: oil, wheat, electricity, etc.
  - Market indices

- The *futures price* is
  - Quoted
  - Drives daily settlement
  - Becomes the settlement price on expiration

Forward price, Arbitrage argument

- F_0 = *forward price* = K that gives the contract zero value at time t = 0
- S_0 = *spot price* today of the underlier
- r = *risk free rate*, continuously compounded, LIBOR, Treasury, …
- Idealizing assumptions:
  - No transaction costs
  - May transact in either direction at the same price: cash, underlier, contract
  - Interest rate known, constant
  - Interest rate for borrowing = interest rate for lending
  - No taxes
  - Possible to hold the asset without carrying costs (storage, dividends, …)
- Arbitrage: way to make money with zero risk -- assumed not to exist.
- Arbitrage argument: if K != F_0 , there is an arbitrage.

$$F_0 = S_0 e^{rT}$$

- Forward price = futures price (in the simple model)

Interest rates

- Bond:
  - Pay X at time 0,
  - Receive *coupon* payments up to time T
  - Receive *face value* (*principal, par* value) = 1 (convention) at time T.
  - X < 1 is a *discount* bond
  - No coupon payments is a *zero coupon bond*
  - B(0,T) = X for a (default) risk free zero coupon discount bond
  - Risks:
    - Interest rate risk -- interest rates may change, reducing the value of the bond
    - Default risk -- the counterparty may default -- not pay. Ignore for now.
- B(t,T) = amount you pay at time t to get one unit ($1) at time T, t<T.
- B(t,T) known at time t, not before
- If $t_1 < t_2 < T$, and you re-borrow at time t_2, then you pay $B(t_1,t_2)B(t_2,T)$
- If $B(t_2,T)$ unknown at time t, then $B(t_1,t_2)B(t_2,T)$!= $B(t_1,T)$
- Constant known interest rate: $B(t,T) = e^{-r(T-t)}$,
- r = risk free rate. Has units %/unit time (e.g. 3%/year).
- *Yield curve*: r not constant, B(t,T) not known at time 0.

Vanilla Put and Call Options

- The holder has the right to sell (put) or buy (call) at price K.
- Asset (spot) price at time t is S_t .
- European style:
  - Exercise only at time T, the expiration time.
  - Cash flow = payout = $V(S_T)$ at time T
  - $V(S_T) = (K - S_T)_+$ (put)
  - $V(S_T) = (S_T - K)_+$ (call)
- American style:
  - Exercise any time up to time T, the expiration time.
  - τ = exercise time
  - Cash flow = payout = $V(S_\tau)$ at time τ
  - $V(S_\tau) = (K - S_\tau)_+$ (put)
  - $V(S_\tau) = (S_\tau - K)_+$ (call)
  - Early exercise strategy: choose τ
- Exchange traded:
  - The exchange assumes counterparty risk
  - Public published prices and market making
- Over the counter, OTC: private agreements.

Exotic Options

- Not exchange traded
- Lookback: trade at the best price in a given period
- Asian: trade at the average price in a given period
- Knockout (barrier): become worthless if the price leaves a given range
- Digital: $V(S) = 1$ or 0, depending on S
- Log contract: $V(S) = \log(S)$
- Basket: trade a specified portfolio at a specified price

Payout diagram

- For the total payout of a portfolio of one or more European option
- All options have the same payout
- May include puts and calls with different strikes
- May or may not include the cost of buying the options

Lognormal model of future prices

- Normal (Gaussian): $X \sim N(m,v)$
- Z = *standard normal*: $m = 0$, $v = 1$.
- $X = m + v^{1/2} Z$
- *Lognormal* distribution: $Y = Ce^X$
- $E[Y] = Ce^{(m+v/2)}$
- Lognormal stock model: $S_T \sim$ lognormal
  - μ = growth rate in %/unit time (e.g. 15%/year)
  - σ = volatility in %/unit time (e.g. 20%/year)
  - $m = (\mu - \sigma^2/2)T$
  - $v = \sigma^2 T$
  - $S_T = S_0 \exp[(\mu - \sigma^2/2)T + \sigma Z T^{1/2}]$
  - $E[S_T] = S_0 e^{\mu T}$
  - Small T, lognormal is approximately $N(\mu T, \sigma^2 T)$
  - Large T, lognormal is highly skewed toward the downside

$$\Pr(S_T > E[S_T]) \sim C \exp(-\sigma^4 T/4) \text{ --> } 0 \text{ as } T \text{ --> infinity}$$

Lognormal model, good and bad

- "*All models are wrong, some models are useful.*" (George Box)
  - A simple model depending on a small number of parameters
  - Many explicit or semi-explicit solution formulas available
  - Derived from a simple dynamic model, geometric Brownian motion
  - Geometric Brownian motion is the limit of the binomial tree model
  - The lognormal model is motivated by the central limit theorem for the binomial tree model

- "*All models are wrong, some models are dangerous.*" (me)
  - The tail probabilities for stock prices and other assets are far larger than the lognormal predicts
  - Real asset price trajectories do not have constant volatility
  - Option pricing using the lognormal model fails to match actual market prices: volatility *skew* and *smile*.

Black Scholes European option pricing

- The Black Scholes theory is discussed in classes 3, 4, and 5
- It is an arbitrage argument like the forward price argument earlier, but requires a more complicated *dynamic hedging* strategy.
- The conclusion is that the market price of a European option should be its *discounted expected value* in the *risk neutral measure*.

$$\text{Price} = e^{-rT} E_{RN}[\, V(S_T) \,]$$

- Risk neutral measure: replace the expected return rate, μ, with the risk free rate, r.
- We will calculate the integral explicitly in class 4 or 5
- Until then, we can estimate the integral by Monte Carlo
- The result is the *Black Scholes formula* for an option price
  - Put price = $P(S_0, K, T, r, \sigma)$
  - Call price = $C(S_0, K, T, r, \sigma)$
- All parameters known at time 0 except σ.
- Implied vol is the σ value that produces the market option price.