Derivative Securities, Courant Institute, Fall 2010

http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec10/index.html

Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment.

Assignment 10, due November 17

Corrections: (None yet)

1. Download and compile the C/C++ files `cumnor.c`, `randnv.cpp`, and `main.cpp`. This is a simulator of default losses in the following model. Each loan pays a continuous coupon of cdt in time dt either until time T or until it defaults. The defaults are exponential with default intensity λ. The present value of the coupon payment for interval $(t, t + dt)$ is $e^{-rt}cdt$. The simulator (once finished) will calculate the losses from a tranche that starts at `bot` and ends at `top`. For example, the 80% to 90% tranche has `top` $= .9$ and `bot` $= .8$. The first losses do not create losses for the tranche because the only create losses for the riskier tranches. After `bot` is reached, further defaults do not create further losses for this tranche because it is used up.
   (a) Find a formula for the present value of the whole coupon stream until time T.
   (b) Find a formula for the loss that results from stopping payments at time $t < T$.
   (c) In the file `main.cpp`, insert a formula for the total present value of the tranche with n identical loans of one unit each, assuming none of them default.
   (d) In the same file, insert a formula for the loss caused by a default at time td, assuming it effects the present tranche.
   (e) In the same file, put in the one factor model of correlated defaults from the week 10 notes. Use the variable `Z0` on the code to represent the common market factor.
   (f) Explore the results using the histogram program in Excel, in particular
      i. Show that when there are many loans, losses are relatively predictable when defaults are uncorrelated but less predictable when losses are correlated.
      ii. Show that when defaults are correlated, returns from mezzanine tranches can be highly skewed, with a small probability of loss, but large expected losses when they occur. You recognize skew returns from skewness in the histogram, with the median return being significantly higher than the mean, or the median loss being much lower than the mean loss.

2. This assignment works through another derivation of the Black Scholes formula that illustrates the method we will apply to bonds next class. Consider an "economy" that consists of the risk free asset that returns rdt in time dt, the stock $dS_t = \mu S_t dt + \sigma S_t dW_t$, and the universe of European style options on S_t. Here μ is the rate of expected return on the stock and σ is its volatility in the real world. We will use the reasoning in the week 10 notes, and Chapter 27 of Hull.

   (a) Show that this economy satisfies the hypotheses, that there is a single source of noise that drives all price, that the time dt correlation between any two risky assets is one. You may assume that for any option, there is some pricing function that given the price of that option in terms of the stock and time. This is a bit of a tautology, but is sets the stage for what follows.

   (b) What is the expected rate of return in the re-weighted measure in which S_t is the numeraire?

   (c) Suppose W_t is the drift free Brownian motion in that re-weighted measure, so that $dE[W_t] = 0$ in the measure in which S_t is the numeraire. Write a formula for S_T in terms of that W_T and T.

   (d) Let $f(S_t, t)$ be the price at time t of an option that expires at time T. Write an expression for $f(S_0, 0)$ using the fact that $f(S_t, t)/S_t$ is a martingale, so that its expected value at the expiration time T is equal to its value at time $t = 0$. Your formula should involve $V(S_T)$, the payout, S_T, and the spot price S_0.

   (e) Now substitute the formula from part (c) into the formula from part (d) to get a formula for $f(S_0, 0)$ as an expectation over $W_T \sim \mathcal{N}(0, T)$. Show that this is equivalent to the pricing formula (13) from the week 4 notes. For this you will have to put (13) into the present notation, with s representing S_0 and the right left side being what we call here $f(s, 0)$.

   (f) Apply the Ito calculus to calculate $d(f(S_t, t)/S_t)$. Show that the fact that this is a martingale in the martingale measure for S_t (that is, with the correctly adjusted drift from part (b)) is equivalent to the fact that $f(s, t)$ satisfies the Black Scholes PDE.