Derivative Securities, Courant Institute, Fall 2010
http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec10/index.html
Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment.

Assignment 8, due November 3

Corrections: (None yet)

Most of this assignment involves using the code `FEuler.cpp`. Download, compile, and run it. It makes an output file `PutPrice.csv`. Graph the data by highlighting the top three rows and making a scatterplot. If it works, you should get a picture like the one in the file `GraphPutPrice.csv` that is posted. The computed prices should be the same too. In all cases below, use the parameters in the code as posted unless the problem asks you to change them.

1. Modify the program to use the correct values of α, β, and γ. Compute the price of a European style put using enough points that the result has converged. You will need to modify the code, possibly by commenting out the lines that take the max with the intrinsic value. Check that you get the same answer as the Black Scholes formula. This is the first step in *validating* the code.

2. Compute the European put price from part (a) with a sequence $nx = 10, 20, 40, 80$, etc. Here nx is the number of points between $S_{\min}$ and $S_{\max}$. If the results are second order, then the difference between the computed approximation using this code and the Black Scholes price should be reasonably accurately approximated by a C/nx^2. An easy way to look for this is to see whether the error decreases by a factor of 4 (roughly) when you double nx. This is a second validation step.

3. For the American style put, make a plot of the *American premium*, which is the difference between the price of the American style option and the corresponding European style option.

4. Look at the American premium for other American style puts with different strikes (not too far in the money, where options do not trade) and expiration dates. Which ones have particularly large and small premia (the correct plural of premium)?

5. Make plots of Delta and Gamma of the American style put. Estimate the derivatives using appropriate finite differences of the numerical solution. Use a fine enough mesh that there are fairly accurate, even if it takes a while to compute. Which of the curves is continuous at the early exercise boundary? It should be very clear from the plots.

6. Modify the code to use a pure smile state dependent volatility (a *local vol*)

$$\sigma(S) \;=\; \min\left(\sigma_0 + \sigma_2\,(S - S_0)^2\,,\,\sigma_{\max}\right)\;,$$

with $\sigma_1 = .3$, $\sigma_2 = 10^{-3}$, and $\sigma_{\max} = .6$. Make sure to compute the time step using the largest value of σ in the problem. For a European put with $T = 1$, compute the implied vol and compare it to the local vol. Are they the same? Note, it would be hard or impossible to make a binomial tree for this.

7. (not to hand in) With the original out of the box code, find a value of nx that makes the code take about a second. Now double nx and count the seconds.

8. (Something theoretical) Suppose the default intensity is time dependent (has a non-trivial *term structure* and is given by $\lambda(t) = \lambda_0 + \lambda_1 t$. Find a formula for the default probability density $f(t)$.