

Derivative Securities, Fall 2010

Mathematics in Finance Program

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<http://www.math.nyu.edu/faculty/goodman/teaching/DerivSec10/resources.html>

Week 8

1 Credit risk

Counterparty risk is the risk that the terms of a contract will not be honored. In particular, *credit risk* is the risk that a counterparty will not fulfill the obligations of a bond. That means missing a coupon payment or failing to pay the principal at maturity. Because of credit risk, borrowers that are perceived as being more likely to default must pay higher interest rates. The difference between the interest of a particular loan or bond and the same loan to a risk free counterparty is the *yield spread*, or the *credit spread*.

Ratings agencies are companies that offer opinions about the credit-worthiness of *fixed income* securities such as corporate bonds and asset backed securities. They do this by issuing a *credit rating*. Each agency has its own short list of possible ratings, ranging from best to worst. For example, Standard and Poors (S&P) ratings, starting from the best, are AAA, AA, A, AB, etc. In the opinion of S&P, all AA rated bonds have about the same default risk, which is very low, but not as low as AAA bonds.

Here are some yield spread numbers today (Wednesday, October 27, 2010). The 5 year Treasury yield (risk free) is 132 (basis points). Yesterday it was 125 and last Wednesday it was 110. It has gone up in a possible sign of increasing inflation, not that 132 bp is a high rate. But Treasuries are a little below the true risk free rate because (add political/policy statement here), so it is a good idea to add the TED spread, which is about basis points at 5 years. This makes today's rate about $132 + 15 = 147$ bp = 1.47%. The yield on 5 year corporate debt by rating was (as measured by an index)

rating	today	yesterday	last Wednesday
AAA	168	164	147
AA	219	210	194
A	267	262	257

This means, for example, that the credit spread today for AAA 5 year debt is $s = 168 - 147 = 21$ bp = .21%. Yesterday, the same AAA 5 year spread was $s = 164 - (125 + 15) = 24 = .24\%$. These spreads are narrow, but the spread was 14% bigger yesterday than today (see below). The credit spread for AA today is $219 - 147 = 72 = .72\%$. This may not seem like a large spread, but it is more than three times as large as the AAA spread. The A spread is a huge 1.2%, which is six times the AAA spread.

There is a fundamental difference in the legal status of a stock and a bond of the same company that is related to what happens upon default. A stock represents part ownership in the company. A bond is a debt of the company to bondholders, who are not owners. A bond represents a debt of the shareholders to the bondholders. The total net value of a company is: net value = assets – liabilities. *Assets* refers to things the company owns, such as buildings, contracts, etc. *Liabilities* are bonds and other debts.¹ By the time a company defaults, its net value usually is negative. When a company becomes bankrupt (declares bankruptcy), its stock is worth zero and stops trading. This represents the understanding that the owners of the company, the stockholders, actually own nothing. Recent big defaults include General Motors, and a few years ago, Enron.

In the formal process of bankruptcy, the assets of the company are converted to cash and distributed to the bondholders and other debt holders. This process is *recovery*. The percentage of recovery is R . One can use $R = .5 = 50\%$ as typical, although much lower values, including $R = 0$, are possible. The amount of loss upon default is $1 - R$.

2 Default intensity model

The simplest model of default is the *Markov jump to default* model. In this model, there is a *default intensity*, λ . Let T be the default time. The probability that $T \in (t, t + dt)$, given that $T > t$ is λdt . This is a *two state* model in that there are two possible states: defaulted or not defaulted. The model is *Markov* because knowing the state at time t is all the information about the past that is relevant for predicting the future. In this case, that means that the probability of defaulting at a time $t' > t$ given that $T > t$ depends only on $t' - t$ and not on t otherwise. The model is that the company does not decline into default, it simply jumps to default without warning.

Many people in the class will recognize this T as an *exponential* random variable with *rate parameter* λ . Let $f(t)$ be the probability density of T , so that $f(t)dt = \Pr(t \leq T \leq t + dt)$. Let $G(t) = \Pr(T \geq t) = \int_t^\infty f(t')dt'$. Note that² $G'(t) = -f(t)$. Now a simple Bayes' rule calculation gives

$$\begin{aligned} \lambda dt &= \Pr(T \in (t, t + dt) \mid T > t) \\ &= \frac{\Pr(T \in (t, t + dt) \text{ and } T > t)}{\Pr(T > t)} \\ &= \frac{\Pr(T \in (t, t + dt))}{\Pr(T > t)} \\ \lambda dt &= \frac{f(t)dt}{G(t)}. \end{aligned}$$

¹Don't worry, this will not become a business school class.

² G is not quite the cumulative distribution function, which is $F(t) = \int_0^t f(t')dt'$.

From this we get $G'(t) = -\lambda G(t)$. Of course, $G(0) = \Pr(T \geq 0) = 1$, so this implies that $G(t) = e^{-\lambda t}$ and $f(t) = \lambda e^{-\lambda t}$.

The default intensity model could be a model for the real world process or a pricing model for the risk neutral process. You can talk about historical default rates that you get from corporate bond defaults in the past. You also can talk about implied default rates that are determined by the yield spread. These implied default rates are not the market's opinion of future default rates, but the market's reaction to its opinion about future defaults. For that reason, you should not use the implied λ as a prediction of future defaults. Instead, the implied λ depends on future defaults and the overall level of risk aversion in the market.

If the risk neutral pricing model is exponential default with parameter λ , then λ is determined by matching the expected return from a bond with default rate λ and recovery R to the yield spread. The result is (more on the derivation below) that the implied λ is given by

$$\lambda(1 - R) = s. \tag{1}$$

You can understand this formula by imagining a very large portfolio of independent but otherwise identical bonds. In a small amount of time, dt , λdt of the remaining bonds will default, causing a loss of $\lambda(1 - R)dt$ times the value of the portfolio. At the same time, the bonds give an excess return above risk free bonds of sdt , again multiplied by the value of the portfolio. Setting these equal gives (1).

Let us use (1) in the form

$$\lambda_{\text{implied}} = \frac{s}{1 - R}$$

with the data above. Supposing 50% recovery makes it simply $\lambda_{\text{implied}} = 2s$. So, a yield spread for AAA of .21% gives $\lambda_{\text{implied}} = .42\%$. This means that over the course of 5 years, roughly³ 2.1% of the bonds will default. A similar calculation using the AA value $s = .74\%$ gives an implied default probability of 7.4%. Historical default rates are far less than this, as you can learn from Hull.

I said that λ_{implied} is not a market prediction of future defaults, but also changes with market risk aversion. To illustrate this, note that λ_{implied} changed in one day by 14%. It does not seem plausible that one between Tuesday and Wednesday, the estimated number of defaults changes 14%.

³Using the excellent approximation $e^{-\lambda t} \approx 1 - \lambda t$.