Derivative Securities, Fall 2010
draft Syllabus

This is a first class on derivatives, options, contingent claims, and other financial structures and contracts. It discusses pricing and hedging theories together with some basic market models. The mathematical tools of stochastic calculus and partial differential equations are developed as needed. Homework assignments will be theoretical as well as computational (Excel, VBA, simple C++).

This is a tentative weekly syllabus for the 14 weeks of the semester.

1. Forward contracts, arbitrage pricing and the forward price. Futures markets, settlement, and the effect of unknown interest rates. European style puts and calls on equities, payout diagrams, kinds of bets. Lognormal model of equity prices, the effect of option nonlinearity, P&L distributions via simulation.

2. General theory of arbitrage pricing in a discrete one period model. The multi-period binomial model, Delta hedging with the underlier or a forward.

3. Risk neutral probability in path space, reweighting, change of "measure" P and Q measures. Martingale measure interpretation of the binomial model. Calibration of a binomial model using volatility. Market implied probability distribution and pricing of exotic options.

4. The continuous time limit using the central limit theorem. Normal and lognormal random variables revisited. Derivation and interpretation of the Black Scholes formula. Calibration and implied volatility. Describing the implied volatility surface using skew, smile, and term structure. Volatility surfaces for equity and currency options – knowing the difference.

5. Option sensitivity and risk measures – "the Greeks". Assessing risk factors in a portfolio of options. Asymptotic properties of the cumulative normal distribution and the Black Scholes formula.

6. Diffusion processes I, continuous time limits. Random walk and binomial trees converge to ordinary and geometric Brownian motion. Diffusions and functions of diffusions, infinitesimal mean and variance, Ito's lemma. Stochastic integrals and stochastic differential equations (lightly).

7. Diffusion processes II, the backward equation and applications. The Black Scholes partial differential equation and the role of final conditions. Backward in time "dynamics" of value functions, well posed problems. Computer solution of backward equations and their relation to binomial and trinomial trees.

8. Backward equations II. American style options, early exercise, smooth pasting. Barrier options and boundary conditions. Pricing with a local

volatility ($\neq$ implied vol surface) surface. Qualitative properties of the solution of a backward equation – why you can't go forward.

9. Interest rates, the yield curve and its possible shapes. The structures of bonds, coupons, principal, duration. Treasuries and LIBOR rates, bootstrapping. Forward rate agreements and swaps Measures of interest rate risk – duration and convexity.

10. Change of measure, re-weighting of diffusion processes, continuous time martingales. The continuous time no arbitrage argument of Black and Scholes. General one factor models, market price of risk, choice of numeraire.

11. One factor short rate models and the yield curve. The Feynman Kac formula and affine models (lots of math here).

12. Interest rate derivatives such as treasury options and swaptions. Pricing interest rate derivatives using Black's method.

13. Credit risk, yield spreads, market implied default rates. Credit ratings and markov transition models. Merton's model of default and hedging of default risk.

14. Bundles and structured products. Default correlation, copulas and implied correlation.