

Derivative Securities, Courant Institute, Fall 2006
Assignment 1, due September 13 [Update: due September 20]
Update (September 19): question 3f fixed

Important: Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment. In particular, problem 5 may change or be eliminated.

1. On day t , the exchange rate between US dollars and Euros was 1.22 \$/Euro. The price of a US 180 day Treasury bill was \$99.48 per \$100 face value. One could buy a payment of 100 Euros in 180 days for 99.46 Euros.
 - (a) What was the theoretical 180 day forward exchange rate?
 - (b) Suppose the market 180 day forward exchange rate was \$1.21 per Euro. Describe the risk free strategy for making money in this market. How many dollars would it have gained for a contract size of 100 Euros?
2. The present price of a stock is 50. The market value of a European call with strike 47.5 and expiration in 180 days is 4.375. The cost of a risk free dollar 180 days hence is $B(0, 180) = .9948$. Show that a price of 1.450 violates put/call parity. Describe how to make a profit with no risk from these prices.
3. For each of the following portfolios, draw the expiry payout diagram and explain what view of the market holding this portfolio expresses. Include a (discounted) price of the option to indicate that the portfolio loses money if the options expire out of the money.
 - (a) Long one call and one put, both with strike price K (a *straddle*).
 - (b) Short one forward and long two calls, all with strike K (also a *straddle*, why?)
 - (c) Long one call and two puts, all with strike K (a *strip*).
 - (d) Long one call with strike K_2 and one put with strike K_1 , with $K_2 > K_1$ (a *strangle*).
 - (e) Long one call with strike K_1 , and one with strike K_2 ($K_1 < K_2$) and short two calls struck at $K = (K_1 + K_2)/2$ (a *butterfly spread*).
 - (f) Describe how to create the butterfly spread payout with only the underlying asset and long positions in puts and calls.
4. Show that the price of a vanilla European call or put is a convex function of the strike price. *Discussion:* A function $f(K)$ is a convex function of K (in the interval $[K_0, K_3]$) if

$$f(\alpha K_1 + (1 - \alpha)K_2) \leq \alpha f(K_1) + (1 - \alpha)f(K_2) , \quad (1)$$

whenever $0 \leq \alpha \leq 1$ (and $K_0 \leq K_1 \leq K_2 \leq K_3$). If $f(K)$ is twice differentiable as a function of K , this is the same as $f''(K) \geq 0$ (for $K_0 < K < K_3$). We can check (1), say, for $\alpha = 1/2$, by comparing the payout diagram for the portfolio:

$$\frac{1}{2}(\text{call at } K_1) + \frac{1}{2}(\text{call at } K_2)$$

to the payout of the single put struck at $\frac{1}{2}(K_1 + K_2)$ (draw the diagram). The argument for other values of α and for puts is similar.

Some options dealers post a curve of options prices at which they are willing to buy and sell as functions of strike and maturity. If the curve comes from interpolation, they must take care that the interpolated curve is convex, or the lack of convexity is less than the spread. Otherwise they can be “arbed” out of business. This has happened.

5. (An exercise in web surfing, please collaborate on the data hunting). On a given day, find

- The Chicago Mercantile Exchange closing price for the December 2006 S&P500 future (standard, not mini).
- The closing level of the S&P500 index itself.
- the appropriate LIBOR rate for about 90 days (get the number of days right). You may have to interpolate between two LIBOR quotes.
- The effective dividend yield rate for the S&P500.

From this data, calculate the effective cost of borrowing and shorting the S&P500 index for that period, assuming the futures price is the forward price.