Derivative Securities, Courant Institute, Fall 2006
Assignment 2, due September 27 [Update: typos fixed Sept. 21, 22]

Important: Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Derivative Securities) before doing any work on the assignment. In particular, some questions may be deleted or added depending on how much material we cover in class.

1. Consider the extreme binomial tree with $u = 2$, $d = \frac{1}{2}$, a spot price $S(0) = 4$ and four time periods called $t = 0, 1, 2, 3$ respectively. Set the risk free rate to zero. Consider hedging a European style call struck at $K = 4$.

   (a) Draw the tree with stock prices and option prices drawn in at all the nodes.

   (b) How many units of money does it take at time zero to reproduce the call payout at $t = 3$? How much of that goes to stock and how much goes to cash.

   (c) Let uud represent the path $S(0) = 4$, $S(1) = 8$, $S(2) = 16$, $S(3) = 8$. Indicate the stock and cash positions at each stage of this path and how many units of stock should be bought or sold in rebalancing.

   (d) Do the same for the path udu. Note that both paths end with a portfolio worth 4 units though the intermediate portfolio values are different.

2. The current stock price is 100 and the volatility is 30%/year. The risk free rate is 6%/year. Consider a one year European call option on this stock with strike price 100 and the option expires in one year.

   (a) Divide the one year period into two six month intervals and use the binomial tree with

   $$u = e^{[(r-\sigma^2/2)\delta t + \sigma\sqrt{\delta t}]} \ , \quad d = e^{[(r-\sigma^2/2)\delta t - \sigma\sqrt{\delta t}]} \ .$$

   Calculate the risk neutral probabilities and show that the option value is 13.65.

   (b) Suppose the initial market price of the option is 14.675 ($= 14\frac{7}{8}$). Assume that the market follows the binomial tree and explain in detail how to take advantage of the market price to produce a risk free profit.

3. Use Stirling's approximation $n! \approx \sqrt{2\pi n}\, e^{-n} n^n$ (for large n) to approximate the binomial probabilities

   $$f(n, k, p) = \binom{n}{k} p^k (1-p)^{n-k}$$

by a gaussian when n is large. The mean is $k^* = np$. Express the answer in terms of $l = k - np$, the deviation from the mean. Explain in detail, including factors of $\sqrt{2\pi n}$, how this agrees with the result from the central limit theorem.

4. Suppose we have an option whose payout depends on a pair of stocks. We want to price using an analogue of a binomial tree, taking into account at each time period that S_1 may go up to $u_1 S_1$ or down to $d_1 S_1$ and similarly with S_2 and u_2 and d_2.

   (a) How many different states of the world would there be after one and after two time periods?

   (b) We want to hedge using both underlying stocks, which we can trade freely as in the Black Scholes theory. Explain whether risk free hedging provides a unique price in this model. Comment on the generality of the binomial tree pricing model.