Ordinary Differential Equations

Homework 1, Revised

Given: September 8
Due: September 13

Section 2.1, problems 14, 18, 30.
Section 2.2, problems 1, 3, 6.
Section 2.3, problems 1 (Drops or streaks of dye are used to visualize fluid flows but once the dye is thoroughly mixed you have to start over with clean water.), 8. You will have to read Section 2.3 on your own to do these problems. Give it a serious try but stop if you just can't get the problems.
Extra problem:

A signal

$$u(t) = A\sin(\omega(t - t_0)) \tag{1}$$

is a sine wave with amplitude A, frequency[1] $f = \omega/2\pi$, and phase offset t_0.

a. Sketch the graph of $u(t)$ for $0 \le t \le 3\pi$ with $A = 1.5$, $\omega = 2$, and $t_0 = 1$. Indicate the geometric significance of the amplitude, period ($= 1/f$), and phase offset on the graph.

b. Suppose

$$u(t) = B\sin(\omega t) + C\cos(\omega t)\ . \tag{2}$$

Show how to write u in the form (1). Hint: use the angle sum formula to write (1) in the form (2) and use this formula to identify B and C in terms of A and t_0. Then reverse this to get A and t_0 in terms of B and C. One of the formulas will be positively Pythagorean.

c. Write a formula for the large time asymptotic behavior or the solution of

$$\dot{v} = -kv + \sin(\omega t)\ . \tag{3}$$

Hint: it does not matter what initial condition you use. For the equation (3), the large time behavior does not depend on the initial condition.

d. Identify the amplitude and phase of this signal as a function of k and ω. What do you think should happen when k is very large or very small?

[1] If t is measured in seconds, then the units of ω and f will both be 1/sec., and f will be "cycles per second".