Ordinary Differential Equations

Homework 5

Given: October 6
Due: October 11 (note: Tuesday, not Thursday)

The ansatz method for finding particular solutions of specific differential equations with forcing: $\ddot{x} + p\dot{x} + qx = f(t)$. If $f(t) = e^{rt}$ (a real or complex exponential) and r is not one of the roots r_1 or r_2, then try $x(t) = Ae^{rt}$. If r is one of the roots, try $x(t) = Ate^{rt} + Be^{rt}$. If $f(t)$ is the real or imaginary part of a complex exponential (i.e. sine or cosine times a real exponential), then solve with f being the complex exponential and take the real or imaginary part. If $f(t)$ is a polynomial, try $x(t)$ a polynomial of the same degree. If $f(t)$ is an exponential times a polynomial, try x also such a function.

1. Find particular solutions for the following:

a. $\ddot{x} - 2\dot{x} + 3x = e^{-2t}$.

b. $\ddot{x} + 2\dot{x} - 3x = e^{3it}$.

c. $\ddot{x} + \dot{x} - 2x = \sin(t)$.

d. $\ddot{x} - \dot{x} - 3x = e^{-t}$.

e. $\ddot{x} + 4x = t\cos(t)$.

f. $\ddot{x} + 4x = \cos(2t)$.

g. $\ddot{x} - \dot{x} - 3x = t^2 + 1$.

h. $\ddot{x} + 4x = t\cos(2t)$.

2. Suppose x_1 satisfies $\ddot{x}_1 + p\dot{x}_1 + qx_1 = f_1(t)$ and x_2 satisfies $\ddot{x}_2 + p\dot{x}_2 + qx_2 = f_2(t)$. Show that $x_1 + x_2$ is a solution of $\ddot{x} + p\dot{x} + qx = f_1(t) + f_2(t)$. Use this to find a solution of

$$\ddot{x} + 4x = t + \sin(t) \tag{1}$$

in the following steps: (i) identify the two forces $f_1(t)$ and $f_2(t)$, (ii) find corresponding particular solutions $x_1(t)$ and $x_2(t)$, (iii) check that $x(t) = x_1(t) + x_2(t)$ satisfies (1). This is the principle of "linearity" or "superposition".

3. Section 3.6, problems 1, 2, 3, 5 (linearity), 13, 17.

4. Section 3.8, problems 6, 14, 16 (easier using complex exponentials).