Ordinary Differential Equations
Homework 8

Given: November 3
Due: November 8

1. Section 7.1, #1, 6, 8 (We will do circuits next week.).

2. Section 7.2, (Do but do not hand in # 1-13), to hand in: #22, 23, 25.

3. Let $f_1(t)$ and $f_2(t)$ be two functions of t. We say a function $x(t)$ is in U if there are constants, a and b, with $\ddot{x} + x = af_1 + bf_2$. (Hint: x has $a = a_1 + a_2$, etc.)

   (a) Show that U is a vector space, that is, check that if $x_1 \in U$ and $x_2 \in U$, then $kx_1 \in U$ for any complex number k, and $x_1 + x_2 \in U$. This boils down to more superposition.

   (b) The dimension of U is 4 (we will see how to know this in a few weeks). Find a basis for U when $f_1(t) = 1$ and $f_2(t) = t$. (Hint: two general and two particular).

4. For the system (1) on page 356 assume $m_1 = m_2 = 1$ and $k_1 = k_2 = k_3 = 1$. Look for solutions of the form

$$x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = e^{i\omega t} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = e^{i\omega t} u(t) \ .$$

   Here u_1 and u_2 are constants that do not depend on time. Do not write the equations in the form of a first order system of four equations.

   (a) Find a quadratic equation for $\lambda = \omega^2$. This equation happens to have two positive roots, so there will be four real values of ω.

   (b) Show that the real and imaginary parts of $x(t)$ also are solutions.

   (c) For each λ, find the corresponding u. These are called *normal modes of vibration*.

   (d) Show that one of the modes corresponds to the masses moving toward or away from each other at the same speed while the other has the masses moving parallel to each other so that the distance between them does not change.

   (e) Since the parallel motion does not engage the k_2 spring, can you guess which mode vibrates faster?