Ordinary Differential Equations

Homework 9

Given: November 10
Due: November 15

1. Section 7.1, #19, 20, 21.
2. Section 7.3, #1, 2, 8, 9, 15, 16, 24.
3. Section 7.5, #5 (by hand without computer), 11.
4. Consider the second order differential equation for a two component column vector

$$\ddot{x} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \dot{x} + \begin{pmatrix} 0 & -4 \\ 2 & -4 \end{pmatrix} x \, . \qquad (1)$$

An exponential solution takes the form $x(t) = e^{rt}\xi$, where ξ is a two component column vector.

   (a) For a general equation $\ddot{x} = A\dot{x} + Bx$, write the equation in terms of matrices A and B and the vector ξ and the number r that we have to solve to find exponential solutions. What is the matrix that has to be singular in order for there to be $\xi \neq 0$? Hint: it involves matrices multiplied by r and r^2, one of them being the identity matrix.

   (b) Write this matrix for the specific problem (1).

   (c) Calculate the determinant of this matrix, which is a polynomial of degree 4 in r.

   (d) Show that this polynomial factors into a product of quadratics one of which is $r^2 + r + 4$. Find the other factor.

   (e) List all the numbers, r_1, r_2, r_3, and r_4, that correspond to exponential solutions of (1).

   (f) What kind of behavior do they represent? (growth/decay, simple/oscillatory)

   (g) Find the ξ corresponding to $r = 1 + i$.

   (h) Take the real part of $e^{rt}\xi$ from part g to find a real solution of (1).

5. In each case there are three elements of the vector space of functions of t. Either show that the functions (vectors) are linearly independent or find a linear combination $0 = c_1 f_1(t) + c_2 f_2(t) + c_3 f_3(t)$.

   (a)

$$\begin{aligned} f_1(t) &= t(t-1) \\ f_2(t) &= (t-2)(t-3) \\ f_3(t) &= (t+2)(t+3) \end{aligned}$$

(b)

$$\begin{aligned} f_1(t) &= \sin(t) \\ f_2(t) &= \sin(2t) \\ f_3(t) &= \sin(3t) \end{aligned}$$