Ordinary Differential Equations, Fall 2005, NYU
Practice for quiz 1.

Information. The actual quiz will follow these rules. Please read them and think about your quiz strategies now.

- Put all answers in the exam book. Do not hand in the question sheet.
- Do not guess. You will receive a few points (depending on the question) for giving no answer. If your answer is completely wrong, you will get zero points. If you do not want to answer a part of the question, state which part you are not answering. For example: "The solution is ... but I don't know how to find the blowup time."
- Give only one answer. You will lose points for incorrect statements even if they are irrelevant to the question or if you also give a completely correct answer. Cross out anything you feel is wrong.

Questions

1. Find the general solution of $t^2\dot{x} = x + 1$. (Hint: integrating factor)
2. Find $y(x)$ that satisfies $\dfrac{dy}{dx} = x^2y^2$, $y(0) = 3$. At what x value does this function "blow up"?
3. Without solving the initial value problem $\dot{x} = x^3 - 3x^2 + 2x$, $x(0) = \frac{1}{2}$:
   (a) Give the fixed points.
   (b) Determine which fixed points are stable and which are unstable.
   (c) Determine $\lim_{t\to\infty} x(t)$.
4. Find all complex numbers $z = x + iy$ with $z^3 = 8$. Write the answers in the form $z = x + iy$ with $x = \mathrm{Re}(z)$ and and $y = \mathrm{Im}(z)$ determined. (Hint: write $z = re^{i\theta}$ and find all possible r and θ values. Remember that different θ values can correspond to the same z.)
5. Use complex exponentials to calculate $\int_0^\infty e^{-2t}\cos(t)dt$. Do not take the real part until the very end.
6. Find the exponential solutions of $\ddot{x} + 2\dot{x} + 5x = 0$. If the solutions are complex, also give the real part and verify using Calculus I methods only that the real part satisfies the differential equation. Classify the behavior of each solution as one of the following:
   - simple exponential growth
   - simple exponential decay
   - oscillation with exponential growth
   - oscillation with exponential decay