Differential Equations, Spring 2026
https://math.nyu.edu/∼goodman/teaching/DifferentialEquations2026/DifferentialEquations.html

Assignment 1, due January 27, 2pm

Last revised (Extra problem, special request) Thursday, January 22

About homework assignments

- Upload one PDF file with homework solutions to the Brightspace page for the appropriate assignment.
- You may write assignments on paper then photograph or scan them. If you do that, please collect all the images into a single pdf file to upload. You may use handwriting or typing on a tablet and upload it in pdf format. You may use LaTeX, but this is not encouraged because LaTeX is less expressive than handwriting and (for me) takes longer to prepare.
- Solutions must be uploaded before class starts on the day assignments are due.
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- Please check the Brightspace forum corresponding to the assignment before you start working on the assignment and from time to time while you are working on it. There may be questions, comments, or (alas!) corrections that will help you.
- Please post any comments or questions or possible corrections on the Brightspace forum for the assignment.
- Please email the instructor director directly with personal matters including requests for a homework deadline extension.
- Be follow the **academic integrity policies** that apply to this class as explained on the **class web page**. In particular, do not submit solutions prepared by AI tools.

The assignment

Pages 33 and 34 Draw the direction fields by hand, not using software graphics tools

- # 1
- # 6 (harder)
- # 13, see supplementary notes on integrals like $\int e^{at}\sin(bt)\,dt$.
- # 20

Pages 33 and 34 Do not use software tools for graphing.

- # 7
- # 14
- # 25 (Do not use a computer numerical tool. Try reasoning with differentials, which is a variant of the approach suggested in the book.)

Pages 49 to 53 Do not use software tools for graphing.

- # 6 (The numbers come from an earlier time when interest rates were higher and prices were lower.)

- # 16 (The **N** label means that you should use a computer or calculator to "crunch" the numbers at the end. It's just a few arithmetic operations.)
- 25 (Do not use a computer numerical tool. Try reasoning with differentials, which is a variant of the approach suggested in the book.)

Extra problem. *Rate equations* are models used to describe the progress of chemical reactions. A generic binary chemical reaction is a molecule of species A combining with a molecule of species B to form a molecule of species AB. This is written $A + B \to AB$, or sometimes $A + B = AB$ (to indicate that the reaction can go both ways, which we will ignore). X_A and X_B are the "amounts" of species A and species B, by which we mean the number of A molecules and B molecules respectively (chemists call this *molar* concentration). If $R\,dt$ is the amount of reaction in a time interval dt, then $X_A \to X_A - R\,dt$ and $X_B \to X_B - R\,dt$ in that time interval. A common model of *reaction rate kinetics* is to suppose that one A molecule combines with one B molecule whenever the "find" each other. In a "well stirred" tank the probability of any particular A molecule finding a B molecule is proportional to the number of B molecules. The number of individual reactions, then, is proportional to X_A, X_B and dt. That is, $R\,dt = kX_AX_B\,dt$. Suppose $X_A(0) = 2$ and $X_B(0) = 1$ (in some units). Formulate and solve a differential equation to find a formula for $X_A(t)$. *Hint.* There are many ways to do this, but all of them lead to the same formula for $X_A(t)$. You could write a differential equation $\dot{X}_A = \cdots$ in terms of X_A and X_B. If you do that, you also need a formula for $X_B(t)$ in terms of $X_A(t)$ and the initial values. You also might write an equation of the form $\dot{X}_{AB} = \cdots$. That has the advantage that the initial condition is simpler: $X_{AB}(0) = 0$.

Suppose A is the *fuel* and B is the *oxidizer*, AB is the (uninteresting) *reaction product* and the reaction is *burning*. Suppose you want enough oxidizer to burn all the fuel. You could use just enough oxidizer to make this happen, which chemists call *stoichiometric*,[1] and is $X_A(0) = X_B(0)$ in this case, or you could use more oxidizer than this minimum. Let T be the time it takes to burn 99% of the fuel. Show that T can be much longer for stoichiometric initial conditions than if you start with more oxidizer than fuel.

Special request. Just before you upload, please estimate the time you spent on this class this week (in hours) in the following activities

- Reviewing class notes
- Reading the textbook
- Reading supplementary notes
- Solving and writing up exercises
- Finally, if you found helpful (to you) online materials, please include a link or a URL.

[1] Chemical reaction systems that are more complicated than this one step binary reaction have stoichiometric ratios that are more complicated than one to one. For example, combining hydrogen (fuel) and oxygen (oxidizer) to make water (reaction product) has stoichiometric ratio two to one because it takes two hydrogen atoms and one oxygen atom to make one water molecule.