

## 30 minute Quiz

### About the quiz

- You may bring one “cheat sheet”. This is one piece of paper you may refer to during the quiz where you can write anything you want. No other materials such as notes, books, computer, phone, networked watch, etc. are allowed. They must be put away and inaccessible during the quiz.
- Write all answers in the answer book (“blue book”) you will be given. The question sheet will not be handed in or graded.
- Points will be deducted for anything you write that is wrong even if you also give a correct answer. Cross out anything you think is wrong.
- Guessing is discouraged. You will get more points for not answering a question than for giving a wrong answer.
- The quiz will be thirty (30) minutes at the beginning of the recitation on February 20.
- Explain your answers with a few words. True/false answers without explanation may get no points. Calculations without explanations, even if correct, may not get full credit. For True/False questions: if it’s true, say why in a few words (not a formal proof). A mathematical statement is false if there’s a counterexample. For example, “All prime numbers are odd” is false because 2 is a counterexample. If the statement is “All prime numbers are odd”, you can answer: “no, 2 is prime and even”. You probably would get full credit for saying: ”no, 2”
- There will be some True/False questions that should be quick to answer and 4 questions that involve calculations that should take around 4 minutes each, on average.
- The practice questions below are to let you know the kind of questions that might be asked. The actual quiz will may be different, but in a similar spirit.

### True/False

1. If  $\dot{x} = f(x)$  and  $x_0$  is a critical point (stationary point) and  $f'(x_0) > 0$ , then there are solutions  $x(t)$  with  $x(0) \neq x_0$  that converge to  $x_0$  as  $t \rightarrow \infty$ . You may explain your answer with a picture.
2. If  $\ddot{x} + a\dot{x} + bx = 0$ , with  $b < 0$ , and  $x = e^{rt}$  is a solution then  $r$  is real.
3. If  $f(x)$  and  $g(x)$  are globally Lipschitz, then  $h(x) = f(x) + g(x)$  is also globally Lipschitz.
4. If  $x_1(t)$  and  $x_2(t)$  are both solutions of  $\ddot{x} = -kx$ , and if  $x_1(0) < x_2(0)$ , then  $x_1(t) < x_2(t)$  for all  $t > 0$ .

### Full answer questions

1. Suppose  $\dot{x} = tx + 1$  for  $t \geq 0$  and  $x(0) = 0$ . Find a formula for  $x(t)$ . If the formula involves an integral, do not “work” the integral but leave the integral in your final answer formula.
2. Suppose  $z(t) = (1+i)e^{3it}$  and  $x(t) = \text{Re}(z(t))$ . Find a formula of the form  $x(t) = A \cos(\omega(t-t_0))$ . Find formulas for  $A$ ,  $\omega$ , and  $t_0$ .

3. Find a formula for  $x(t)$  so that

$$\ddot{x} = -x - \dot{x} \quad , \quad x(0) = 1 \quad , \quad \dot{x}(0) = 0 \quad .$$

4. Suppose

$$dx = -a \left( 2 + \sin(x^2) + \sqrt{1 + x^2} \right) dt \quad , \quad x(0) = 1 \quad , \quad a > 0 \quad .$$

Let  $T_a$  be the time  $x(t)$  crosses the  $x$  axis. That is,  $x(T_a) = 0$ .

- (a) Show that there is exactly one such crossing time. This can be done with a picture and an explanation of the relevant properties of  $f(x) = 2 + \sin(x^2) + \sqrt{1 + x^2}$ . Do not look for a formula for  $x(t)$ , which is impossible to find.
- (b) Show that  $T_a$  is proportional to  $\frac{1}{a}$ . This can be done by normalizing/rescaling to set  $a = 1$ . Do not look for a formula for  $C$  so that  $T_a = C\frac{1}{a}$ . This is impossible to find.