Quiz practice

About the quiz

- You may bring one "cheat sheet". This is one piece of paper you may refer to during the quiz where you can write anything you want. No other materials such as notes, books, computer, phone, networked watch, etc. are allowed. They must be put away and inaccessible during the quiz.
- Write all answers in the answer book ("blue book") you will be given. The question sheet will not be handed in or graded.
- Points will be deducted for anything you write that is wrong even if you also give a correct answer. Cross out anything you think is wrong.
- Guessing is discouraged. You will get more points for not answering a question than for giving a wrong answer.
- The quiz will be twenty (20) minutes at the beginning of the recitation on February 20.
- Explain your answers with a few words. True/false answers without explanation may get no points. Calculations without explanations, even if correct, may not get full credit. For True/False questions: if it's true, say why in a few words (not a formal proof). A mathematical statement is false if there's a counterexample. For example, "All prime numbers are odd" is false because 2 is a counterexample. If the statement is "All prime numbers are odd", you can answer: "no, 2 is prime and even". You probably would get full credit for saying: "no, 2"
- There will be some True/False questions that should be quick to answer and 4 questions that involve calculations that should take around 4 minutes each, on average.
- The practice questions below are to let you know the kind of questions that might be asked. The actual quiz will may be different, but in a similar spirit.

True/False

1. A solution to a linear ODE may blow up in finite time.
2. A second order linear equation $\ddot{x} = a\dot{x} + bx$ always has a two dimensional vector space of solutions.
3. Depending on the parameters $m > 0$, $\gamma > 0$ and $k > 0$, solutions of the spring/mass/friction system always decay to zero as $t \to \infty$ but they be oscillatory or not depending on the values of the constants.
4. If any solution of an ODE blows up in finite time, then every solution does.
5. If $\dot{x} = f(x)$ with $f(0) = 0$, then $x(t) = 0$ is a solution. Any other solution has the property that $x(t) \to 0$ as $t \to \infty$.
6. The function $f(x) = \sin(x)$ is globally Lipschitz.
7. The function $f(x) = e^x$ is locally Lipschitz but not globally Lipschitz.

8. Any continuous function is at least locally Lipschitz.

9. There is a first order scalar ODE $\dot{x} = f(x)$ that has an oscillatory solution in the sense that $x(t)$ does not blow up in finite time or go to infinity as $t \to \infty$, does not have a limit as $t \to \infty$, but keeps "going up and down".

Full answer questions

1. Suppose $\dot{x} = \frac{1+x^2}{x}$ and $x(0) = 1$. Find a formula for $x(t)$. *Hint.* In the method of separation of variables, let $y = \frac{1}{2}x^2$, so $dy = xdx$ and $1 + x^2 = 1 + \frac{1}{2}y$, so

$$\frac{x\,dx}{1+x^2} = \frac{dy}{1+\frac{1}{2}y} .$$

Then you get a formula for y as a function of t and finally a formula for x as a function of t.

2. Consider the inhomonogeneous initial value problem

$$\dot{x} = -x + t \; , \;\; x(0) = 0 \; .$$

Carry out the following two solution procedures and correct algebra mistakes until the two solutions agree

- The ansatz methods: try $x(t) = a + bt + ce^{-t}$ and find a, b and c.
- Put x on the left and use an integrating factor to get an integral formula for the solution, then find a formula for the integral.

3. Suppose a round (spherical) ball loses volume, dV, in time dt proportional to its surface area and dt. Write an ODE that represents this model. Use your ODE to show that the time until the ball disappears completely is proportional to its initial volume. This is true no matter what the constant of proportionality between dV and Adt is. Recall that the volume inside a sphere of radius r is $V = \frac{4}{3}\pi r^3$ and the surface are is $A = 4\pi r^2$. This gives a formula for the A as a function of V. Be careful with signs: dV is negative if the ball is losing volume.

4. Use Euler's formula for $e^{i\theta}$ to show that

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2} .$$

Square this to get a formula for $\cos^2(\theta)$ in terms of $\cos(2\theta)$.

5. Consider the ODE $\dot{x} = -ax^3 + \cos(\omega t)$, $x(0) = x_0$. Show that this equation may be "normalized" by re-scaling x and t to set $a = 1$ and $\omega = 1$. That is, find constants c, τ, and y_0 so that that the solution has

$$x(t) = c\,y\left(\frac{t}{\tau}\right) \;\; , \;\; \frac{dy}{dT} = -y^3 + \cos(T) \; , \;\; y(0) = y_0 \; .$$

Comment. The relation between the original time variable t and the "normalized" (also "nondimensionalized") time variable T is $t = \tau T$.

6. Find a formula for x that satisfies

$$\ddot{x} = x + 1 \;\; , \quad x(0) = 0 \; , \;\; \dot{x}(0) = 0 \; .$$

Hint. $x(t) =$ constant is a 'particular solution", for the right constant.