Differential Equations

Jonathan Goodman, Fall, 2026

Schedule (tentative)

First order equations

Jan 20, Solving first order scalar equations

- Skim Chapter 1, we will return to points made there throughout the class.
- Sections 2.1 and 2.2
  - Calculus tricks for finding a solution formula
  - Working with differentials, intuition and theory
  - Understanding what a solution formula says about the qualitative behavior of a solution

Jan 22, Modeling with first order equations

- Section 2.3
  - Exponential growth and decay models
  - Nonlinear effects such as saturation and solution-dependent growth rates
  - Continuous time and space models of large but discrete systems

Jan 27, Qualitative behavior, fixed points (upload Assignment 1 by: **2 pm**)

- Section 2.5 (skip Section 2.4)
- Supplementary material on blow-up
  - Example, the logistic equation
  - Fixed points and their stability/instability
  - The "topology" of the real axis with fixed points
  - Finite time blow up

Jan 29, Summary and conclusions

- Section 2.6
- Skim Section 2.7 (We will do this material later)
- Section 2.8 (skim the proof)
  - Exact differentials (integrating factors + calculus tricks
  - The initial value problem, short time vs. global existence
  - Lipschitz continuity, local Lipschitz vs. global Lipschitz

Second order linear equations

Feb 3, Basic math of solutions (upload Assignment 2 by: **2 pm**)

- Section 3.1
- Section 3.2 (skim)
- Section 3.3
- Section 3.4 (skim)
  - Formulation of second order equations and the initial value problem
  - The characteristic equation
  - Complex roots, exponential growth/decay (real part) oscillation rate (imaginary part)

Feb 5, Inhomogeneous equations, forcing

- Section 3.5
- Section 3.6
  - Formulation of the initial value problem with forcing
  - General and particular solutions
  - Guess the answer, "undetermined coefficients"
  - Express the answer as an integral "variation of parameters"

Feb 10, Linear oscillators(upload Assignment 3 by: **2 pm**)

- Section 3.7
  - $F = ma$, spring/mass systems, restoring force, mass, friction
  - The linearized pendulum (not covered in the chapter)
  - Dimensional analysis and scaling of the frequency and damping rate
  - Underdamped and overdamped systems

Feb 12, Forces oscillators and resonance

- Section 3.8 (challenging material, essential for applications)
  - Non-resonant periodic forcing
  - Resonant periodic forcing and linear growth of solutions
  - Resonance peaks and their widths for forced oscillators with friction

Linear systems of equations

Feb 19, Formulation and examples of linear constant coefficient systems

- **30 minute quiz**
- Section 7.1
- Section 7.2
  - Examples, coupled oscillators
  - Examples, electric circuits
  - Review of vectors and matrices, addition and multiplication
  - Formulation: a system of first order linear constant coefficient equations

Feb 24, Linear algebra review, role of eigenvalues and eigenvectors (upload Assignment 4 by: **2 pm**)

- Section 7.3
- Section 7.4 (don't worry about determinants and Wronskians)
- Section 7.5 (one of the most important of the course)
  - Eigenvalues and eigenvectors of matrices
  - Superposition of solutions
  - The vector space of solutions,
  - The solution corresponding to an eigenvalue/eigenvector pair
  - The general solution as a superposition of eigenvector solutions

Feb 26, Complex eigenvalues, growth, decay, oscillation

- Section 7.6 (main focus)
- Section 7.8 (be aware of this but do not focus on it)
  - Significance of the real and imaginary part of the eigenvalue
  - Behavior of solutions
  - Examples involving circuits and mechanical systems
  - Using Python and `numpy` to compute eigenvalues and eigenvectors

March 3, Fundamental solution, impulse response, forcing (upload Assignment 5 by: **2 pm**)

- Section 7.7
- Section 7.9
  - The fundamental solution matrix and matrix exponential
  - Solution with forcing using the fundamental solution
  - A bit about the Laplace transform

Nonlinear equations, phase plane and local behavior

March 5, First order systems of two equation systems

- Section 9.1 (mostly review)
- Section 9.2
  - Linear equations, kinds of behavior
  - Example: nonlinear pendulum,
  - Nonlinear problems, stationary/critical points
  - Behavior near critical points via linearization
  - Phase plane analysis of the nonlinear pendulum

March 10, Larger systems of nonlinear equations (upload Assignment 6 by: **2 pm**)

- Section 9.3
- Section 9.4
- Section 9.6 (skim)
  - Examples: double pendulum, chemical rate equations
  - Critical points and local behavior
  - Global behavior via phase plane, species competition models

March 12, Periodic solutions, limit cycles,

- Section 9.5
- Section 9.7
  - Preditor/prey model via the phase plane
  - Poincaré Bendixson theorem
  - Super-critical bifurcation (lightly)
  - Vander Pol equation

March 24, Chaos (upload Assignment 7 by: **2 pm**)

- Section 9.9
- Supplementary notes
  - Sensitivity of solutions, Liapunov exponents
  - The Lorenz attractor
  - Other examples of chaos

Midterm exam, March 26

Power series in theory and practice

March 31, Power series

- Section 5.1
  - Convergence/non-congergence, "radius" of convergence
  - Examples of Taylor series, distance to complex singularities
  - Term by term differentiation and integration
  - Multiplication and substitution of series

April 2, Power series representation of differential equation solutions

- Section 5.2
- Section 5.3 (lightly)
  - Constructing the series term by term, recurrence relations
  - Assessing the radius of convergence
  - Assessing growth/decay of coefficients, analytical technique

April 7, Singular points of the differential equation, (upload Assignment 8 by: **2 pm**)

- Section 5.4 (pay attention to the goal: finding local power law behavior)
- Section 5.5 (don't read the algebra in detail, see what they're trying to accomplish)
- Section 5.6 (skim)
- Section 5.7 (pay attention to the technique, not the classification of Bessel functions)
  - Express the local solution as a "power law" with power series correcctions
  - Find the behavior of solutions by finding formulas that satisfy the differential equation locally
  - Bessel functions are important for partial differential equations in 2D

Arpil 9 Introduction to numerical computing in Python

- Supplementary notes
  - Computing and plotting solutions given by power series

- Environment, variables, control flow, loops
- `Numpy` arrays
- Plotting
- Goal: create plots like Figure 5.7.1, etc.

Numerical computing

April 14, Time stepping methods

- Section 8.1
- Section 8.2
- Section 8.3 (skim)
- Section 8.5 (important and easy)
  - Numerical time stepping methods for the initial value problem
  - Forward Euler method
  - Quantitative error analysis
  - More accurate multi-stage Runge Kutta methods
  - Time step methods in Python

April 16, Differential equation solving software in Python/`numpy`

- Supplementary notes
  - What fancy software does to reach a specified accuracy efficiently:
    * Variable and high order methods
    * Error estimation
    * Variable and adaptive time step selection
  - Example: double and triple pendulum, convert second order to first order
  - Visualizing solution: movie

Fourier series, heat and wave equations

April 21, Fourier sine and cosine series

- Section 10.1 (pay attention to the goal: finding local power law behavior)
- Section 10.2 (don't read the algebra in detail, see what they're trying to accomplish)
- Section 10.3 (skim)
- Section 10.4 (easy but important)
  - Two point boundary value problems and eigenfunctions for ∂_x^2
  - Fourier sine and cosine series for periodic functions, orthogonality and formulas
  - Examples, piecewise constant and linear
  - Completeness (no proofs)
  - Gibbs phenomenon

Arpil 23 Fourier series and the heat equation

- Section 10.5
- Section 10.6
  - Derivation of the heat equation, energy flux, boundary conditions

- Separation of variables methods
- Qualitative behavior, smoothing, decay of Fourier coefficients

Arpil 28 The wave equation, modes of vibration

- Section 10.7
  - Derivation of the wave equation
  - Wave propagation
  - Wave reflection at boundaryes
  - Modes of vibration
  - Harmonics and superposition