30 minute quiz, October 7

Instructions:

- Explain your reasoning. Points may be subtracted even for correct answers otherwise.
- Cross out anything you think is wrong. Points may be subtracted for wrong answers even if the correct answer also appears.
- Answer each question in the space provided.

Questions:

1. Suppose assets A and S have price 1 today and the prices in the table tomorrow.

	asset	A	S
state			
1		1	2
2		1	3

   What is the price today of an option that pays $V = 0$ tomorrow in state 1 and $V = 1$ tomorrow in state 2. Note that the risk free rate (the return on asset 1) is zero.

   (a) Explain how to replicate asset V with a combination of assets A and S.

   (b) What is the price of asset V today?

2. A continuously compounded account increases the initial investment by a factor e^{tr}. Without compounding, the initial investment increases only by a factor of $1 + rt$. Suppose the interest rate is $r = .05\%$/year and the initial investment is \$100. Estimate the time until the account with compounding is worth \$2 more than the account without compounding. Use Taylor series approximation of the exponential up to the appropriate order.

3. Suppose T is an exponential random variable with rate parameter λ. Calculate the covariance $C_{T,T^2} = \mathrm{cov}(T, T^2)$ You may use the fact that:

$$\mathrm{E}[\,T^n] = \frac{n!}{\lambda^n} \quad \text{for any } n \geq 0$$

4. Suppose X is a Gaussian random variable with mean $\mu = 10$ and variance $\sigma^2 = 4$. Suppose Y is a put on X with strike price $K = 10 = \mu$.

   (a) Draw a graph of the payout Y as a function of X for X in the range $0 \leq X \leq 20$.

   (b) Calculate the expected value of the payout and get a number. You may use the approximation $\frac{1}{\sqrt{2\pi}} \approx .4$, or $\frac{1}{\sqrt{2\pi}} \approx .399$, or $\frac{1}{\sqrt{2\pi}} \approx .39894$ (you choose).

5. Write an `R` script that calculates the sum S below. Do not use the formula $S = \frac{1}{3}n^3 + \cdots$.

$$S = \sum_{k=1}^{n} k^2 \ .$$