Assignment 9, due ??

Corrections: (none yet. See message board)

1. The Black Scholes formula for the market price of puts and calls is

$$
\begin{aligned}
d_1 &= \frac{\log(S_0/K) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\
d_2 &= \frac{\log(S_0/K) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T} && \text{(alternative formula)} \\
P &= Ke^{-rT}N(-d_2) - S_0N(-d_1) && \text{(put price)} \\
C &= S_0N(d_1) - Ke^{-rT}N(d_2) && \text{(call price)}
\end{aligned}
$$

   Use the limits $N(d) \to 1$ as $d \to \infty$ and $N(d) \to 0$ as $d \to -\infty$ to show that the Black Scholes prices for deep in-the-money puts and calls agrees with the price of the corresponding forward contract. Part of this exercise is figuring out what the terms mean and what it means for prices to "agree".

2. The *Delta* of an option, denoted Δ, is the derivative of the options price with respect to the spot price

$$
\begin{aligned}
\Delta_P &= \frac{\partial P(S_0, K, T, \ldots)}{\partial S_0} \\
\Delta_C &= \frac{\partial C(S_0, K, T, \ldots)}{\partial S_0} .
\end{aligned}
$$

   Compute the formula for Δ_C. Hint: Use the chain rule. Write separate formulas for $\frac{\partial C}{\partial d_1}$ and $\frac{\partial d_1}{\partial S_0}$, etc., then put them together. Check that your formula has the right behavior when S_0 is deep in-the-money and deep out-of-the-money. Part of this exercise is figuring out what "the right behavior" is.

3. Repeat exercise 2 (find the derivative, check that it makes sense for large and small K) for the derivative of the call price with respect to the strike price, K.

4. This programming exercise does bisection search to find the implied vol of a given option price. Along the way it introduces some new programming

concepts. There are three `R` files to download. `ImpliedVol.R` is the "main program", which is the script you run from from the `R` command window. It sets up the problem, sets the parameters, and prints the answer. Notice line 7, which is `rm(list=ls())`. This removes all previously defined variables. That way if you run the script twice, the second one will not have variables left over from the first time. If you run the script a second time with the line `K=...` commented out, then `K` will not be defined. Without `rm(list=ls())`, the value of `K` would be left over from the first time.

The mathematical/financial task of this exercise is testing the Black Scholes theory against some market data. You will probably find that the market prices are in rough agreement with the theory. You need to write a function in `R` to evaluate the Black Scholes formula and the derivative of the Black Scholes formula with respect to K. These function definitions should go in the file `OptionPricer.R`. You should put code in the script `Implied Vol.R` to test whether the derivative code you wrote is consistent with the function values. For that, choose a reasonable K and several ΔK values and see whether

$$\frac{\partial f(\cdots, K, \cdots)}{\partial K} \approx \frac{\partial f(\cdots, K + \Delta K, \cdots) - f(\cdots, K, \cdots)}{\Delta K} \ .$$

The left side should be evaluated by the function that evaluates the derivative with respect to K. The right side uses the Black Scholes formula, which you coded.

The next task is to use the bisection search function to calculate implied volatility. For this, you need to find vol parameters σ_a and σ_b so that the implied vol is inside the interval (σ_a, σ_b). You don't have to be fancy here, it's OK to experiment and put in numbers you found to work. The bisection code will print an *error message* if your interval is too small. Look at the code to see how that works. Note that the bisection search code doesn't know much about the function it's working on. That makes the whole code, the collection of functions and the main program, modular.

The final task is to look up near the money option prices for SPX or SPY (you choose) that seem to be very liquid (small bid/ask spread, lots of outstanding interest, lots of trading). Take several put prices with the same expiration and nearby strikes. Calculate the implied vol for one of those prices, then use your derivative function to estimate the price for the other one. Discuss how accurate the result is.

Hand in printouts of the three files. Hand in printouts that illustrate the results you got. Make sure the code is up to reasonable coding standards

- Good use of white space
- Alignment of equal signs, within reason
- Generous and helpful comments (i.e., not `x = y # set x equal to y`)

- Clear helpful variable names, preferably not too long. I like to use the same letter in the code that is used in the formulas. For example, I would call the strike price `K` rather than `strikePrice`.