Numerical Methods II, Courant Institute, Spring 2012
http://www.math.nyu.edu/faculty/goodman/teaching/NumMethII/index.html
Always check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on Numerical Methods II) before doing any work on the assignment.

Assignment 1, due February 9

Corrections: (none yet)

1. One of the problems with high order Adams methods is that you need accurate initial values. Explain how to construct such values using just the first order forward Euler method and Richardson extrapolation. Does this significantly increase the cost of integrating an ODE up to some time T?

2. Calculate the modified equation for the second order Adams Bashforth method. What does it suggest will go wrong in the long run if you apply it to the *harmonic oscillator*, which is the linear ODE for circular motion $\dot{x} = y$, $\dot{y} = -x$.

3. Consider the Adams Bashforth and Adams Moulton methods with p lags. The explicit method has order p and the implicit method order $p + 1$. Write them as

$$x_{n+1} = x_n + \Delta t \left(b_0 f(x_n) + \cdots + b_p f(x_{n-p}) \right) \qquad \textit{(explicit)}$$

and

$$x_{n+1} - \Delta t c_{-1} f(x_{n+1}) = x_n + \Delta t \left(c_0 f(x_n) + \cdots + c_p f(x_{n-p}) \right) \qquad \textit{(implicit)}$$

For the implicit method, you need to solve a nonlinear system of equations to find x_{n+1} at each time step. In implicit Adams Moulton method is derived by integrating the interpolating polynomial over the interval $[t_n, t_{n+1}]$, where the polynomial is order p and interpolates at times t_{n+1}, t_n $, \ldots, t_{n-p+1}$. You can prove (or look in the references for a proof) that the Adams Moulton method is order $p + 1$ rather than order p for the Adams Bashforth method.

The *predictor-corrector* method is a way to get the accuracy of Adams Moulton without solving a system of equations. The *predictor* step is

$$\overline{x}_{n+1} = x_n + \Delta t \left(b_0 f(x_n) + \cdots + b_p f(x_{n-p}) \right)$$

This is an order p accurate prediction of x_{n+1}. The *corrector* step is

$$x_{n+1} = x_n + \Delta t \left(c_{-1} f(\overline{x}_{n+1}) + c_0 f(x_n) + \cdots + c_p f(x_{n-p}) \right) .$$

Show that if the predictor scheme (Adams Bashforth) is order p and the implicit method (Adams Moulton) is order $p + 1$, then the predictor corrector method is also order $p + 1$.

4. **Programming.** This is a relatively simple coding task to do in C++. Write a program that applies the forward Euler method and one that applies the second order Adams Bashforth method for the system of equations

$$\begin{aligned}\dot{x} &= y + \epsilon\left(1 - x^2 - y^2\right)x \\ \dot{y} &= -x + \epsilon\left(1 - x^2 - y^2\right)y\end{aligned}$$

with initial conditions $x(0) = .5$, $y(0) = 0$. For the second order method, you will need to start by predicting x_1 to second order using the forward Euler method (or some other way). For small positive ϵ, the solution slowly spirals out to the circle of radius 1. Write a program that tests the correctness of the code by checking that it does the right thing (approximately) when $\epsilon = 0$. Integrate out to time $t = 2\pi = 2 \cdot 3.1415926535...$. Write a program that uses Richardson extrapolation to verify the theoretical orders of accuracy of the two methods for $T = 2\pi$. Then do some runs for much longer T with $\epsilon = 0$ and verify that the long time behavior of the methods is as predicted by their respective modified equations.

Notes on programming. The programs called for here are very simple from a computer programming point of view. You are not asked to do graphics, for example. Just print out a few numbers and explain how they answer the questions posed above. Please print out and hand in the codes you used. In future assignments we will focus on aspects of coding ignored here – modularity, readability, etc.