**Risk and Portfolio Management with Econometrics**, Courant Institute, Spring 2009
http://www.math.nyu.edu/faculty/goodman/teaching/RPME09/index.html
**Always** check the class bboard on the blackboard site from home.nyu.edu (click on academics, then on the course name) before doing any work on the assignment.

# Assignment 5, due February 25

**Corrections:** (none yet)

1. Verify the formula (7) in the proof of the Lemma.

2. Verify the second step in the proof of the SVD, that $u_3 = Av_3$ is orthogonal to $u_1$ and $u_2$. Note that one of these verifications is a consequence of the lemma.

3. Suppose $A$ is an $n \times m$ matrix with $m \leq n$. Show that

$$\sigma_m \;=\; \min_{\|y\|=1} \|Ay\| \;.$$

   Hint: Try using:

$$\|Ay\|^2 \;=\; (Ay)^t \,(Ay) \;=\; y^t \left(A^t A\right) y \;.$$

4. Show that $\|X - X_k\|_F^2 \;=\; \sum_{j=k+1}^{m} \sigma_j^2$. That is, the error in the optimal rank $k$ approximation is the sum of the neglected singular values (principal components).

5. Show that the Tychanoff regularization formula is the solution to the *penalized* least squares minimization problem

$$\min_b \left( \|Y - Xb\|^2 \,+\, \epsilon^2 \|b\|^2 \right) \;.$$

   Here, $\epsilon$ is the *penalty* parameter that penalizes very large regression coefficients.