Assignment 6.

Given April 8, due April 22.

Objective: To explore numerical methods of ODE's.

We want to compute the trajectory of a comet. In nondimensionalized variables, the equations of motion are given by the inverse square law:

$$\frac{d^2}{dx^2}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{-1}{(x^2+y^2)^{3/2}}\begin{pmatrix} x \\ y \end{pmatrix} \quad .$$

We will always suppose that the comet starts at $t = 0$ with $x = 10$, $y = 0$, $\dot{x} = 0$, and $\dot{y} = v_0$. If v_0 is less than the escape velocity ($1/\sqrt{5}$ here), the comet will move in an elliptical orbit, returning to $x = 10$, $y = 0$ after a time $T(v_0)$. This is the time for a single orbit.

(1) Compute the position and velocity at time $t = 30$ with $v_0 = .2$ using a fixed Δt and the three methods: Forward Euler, second order Adams Bashforth, and four stage fourth order Runge Kutta. Do a convergence study to verify the expected order of accuracy of each of these methods.

(2) Use a fixed time step to compute $T(v_0)$ as a function of v_0 for v_0 in the range $[.01, .25]$. Plot the result. For this, you will need some way to pick an appropriate time step, possibly by doing a convergence study. You will also have to safeguard your program against the possibility that $T(v_0)$ is infinite.

(3) Write an adaptive time step program that uses the fourth order Runge Kutta together with a local truncation error strategy such as the one discussed in class. The adaptive program should much more efficient than the fixed step method when v_0 is small.