# Assignment 7.

Given April 22, due May 6.

**Objective:** To explore basic Monte–Carlo and the central limit theorem.
**Warning:** Monte–Carlo burns computer time. This assignment has been scaled to fit into a recent model Pentium computer or better. If it takes too long on your computer, scale it down a bit.

Let $X$ be the random variable $X = \max(T_1, T_2, T_3)$, where the $T_j$ are three independent uniformly distributed random variables in the interval $[0, 1]$.

**(1)** Make a histogram of $10^4$ independent $X$ values with a bin size .01. The probability density function for $X$ is $f(x) = 4 \cdot x^3$. On the same plot, plot the properly scaled density function, $f$, to see that the empirical histogram and the underlying density agree for large samples.

**(2)** Let $S_n$ be the sum of $n$ independent $X$ samples. That is:

$$S_n = \sum_{k=1}^{n} X_k \quad ,$$

where the $X_k$ are independent (each with its own different independent $T_1$, $T_2$, and $T_3$). Show that the mean value $m_n = E(S_n)$, and the variance, $v_n = E[(S_n - m_n)^2]$ grow linearly with $n$. Do this using $10^4$ samples of $S_n$ for $n = 5j$, $j = 1, \ldots, 20$. Make a plot of $m_n$ as a function of $n$ and one of $v_n$.

**(3)** Make a histogram of the values of $S_{100}$ using about 100 bins scaled so that about 99% of the data is in some bin. On the same graph, plot a Gaussian density function with the appropriate mean and variance. If the Gaussian in properly scaled, it should fit the histogram data rather well. This is an instance of the central limit theorem.

**Extra Credit:** Define $Y_k$ by $Y_1 = 0$ and $Y_{k+1} = .5Y_k + T_k$ where the $T_k$ are independent uniform $[0, 1]$ random variables. Define $S_n$ to be

$$S_n = \#\{k \leq n \mid Y_k > 1\} \quad .$$

Use Monte–Carlo simulations to show that $S_n$ is approximately Gaussian when $n$ is large.