Stochastic Calculus, Courant Institute, Fall 2020
http://www.math.nyu.edu/faculty/goodman/teaching/StochCalc2020/index.html

Six Classes on Stochastic Calculus

Jonathan Goodman, Fall, 2020

Week 1 Brownian Motion, scaling, the heat equation

1. Introduction to the course
2. Homogeneous independent increment processes
3. Discrete time approximations, random walk, scaling
4. Convergence in distribution for paths
5. Gaussian transition densities

Week 2 Strategies and the Ito integral, Ito's lemma

1. Discrete time strategies,
2. Value functions with strategies
3. Strategies on martingales, Doob martingale theorem
4. Strategies in continuous time
5. Ito integral limit, continuous time Doob
6. Examples:
$$X_t = \int_0^t W_s dW_s = \frac{1}{2}\left(W_t^2 - t\right) \ .$$
7. Ito's lemma for Brownian motion

Week 3 Value functions and backward equations

1. Value function $f(x,t) = \mathrm{E}[V(X_T)]$ and backward heat equation
2. Backward heat equation directly and from Ito's lemma
3. Examples: digital and Gaussian payout
4. Hitting probabilities, boundary conditions
5. Integral value functions

Week 4 Diffusion processes and SDE models

1. Infinitesimal mean (drift) and infinitesimal variance (quadratic variation).
2. Example: mean reversion, drift, Ornstein Uhlenbeck
3. Example, geometric Brownian motion
4. Solution formula $S_t = S_0 e^{\sigma W_t} e^{(\mu - \frac{1}{2}\sigma^2)t}$.

5. Ito's lemma for diffusion processes
6. Backward equations for diffusions

Week 5 Hedging, control, Black Scholes.

1. Black Scholes hedging strategy, as understood by Black and Scholes.
2. Binomial tree limit. Black Scholes as understood by CRR.
3. The Black Scholes equation and the risk free process
4. The Black Scholes formula
5. Merton optimal dynamic investment?

Week 6 Change of measure

1. Feynman Kac formula for multiplicative functionals
2. Application to interest rate models
3. Change of measure and likelihood ratio/Radon Nikodym derivative
4. Absolutely continuous and completely singular measures
5. Girsanov change of measure formula

What's missing lots

1. Multi-dimensional diffusions – diffusions with more than one component
2. Forward equation for evolution of probability density
3. Qualitative behavior of PDE, smoothing, maximum principle, ...